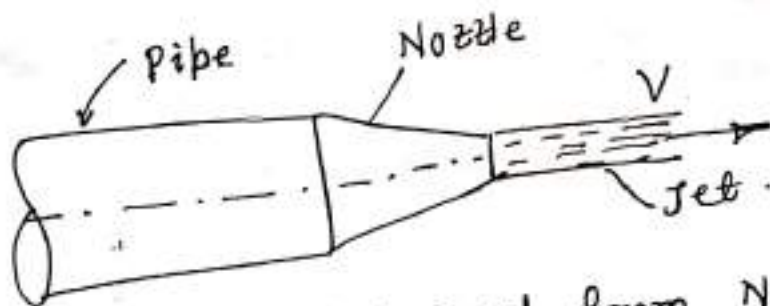


Impact of Jet: ~ A fluid jet is a stream of fluid issuing from a nozzle when a jet of fluid emerges from a nozzle it has some velocity and hence it possesses a certain amount of kinetic energy. If this jet strikes an obstruction placed in its path, it will exert a force on the obstruction. This impressed force is known as impact of the jet and it is designated as hydrodynamic force.



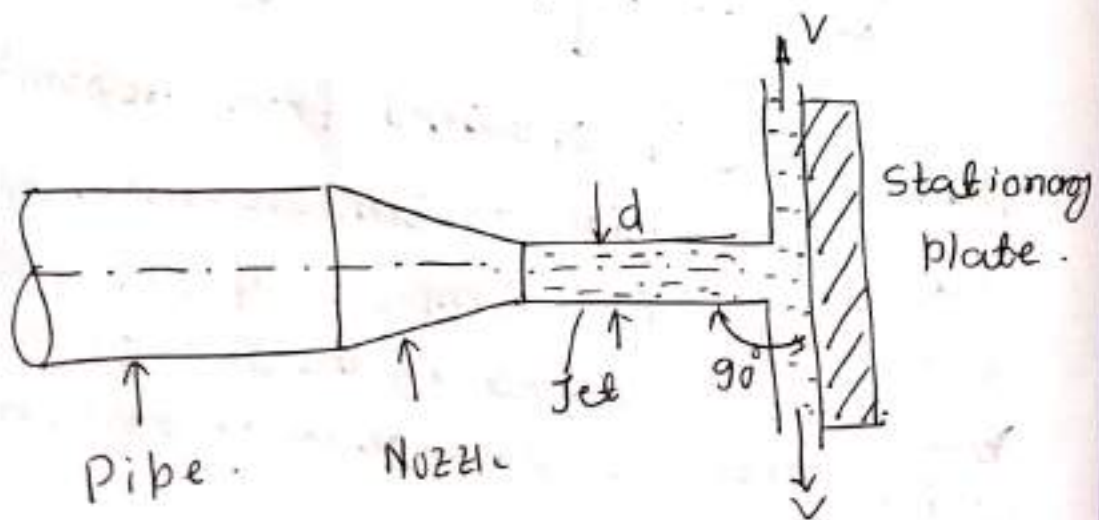
This force is obtained from Newton's 2nd law of motion or impulse-momentum equation. Thus impact of jet means the force exerted by the jet on a plate which may be stationary or moving.

In this chapter, the following cases of the impact of jet will be considered.

- (i) Force exerted by the jet on a stationary vertical flat plates and
- (ii) force exerted by the jet on a moving flat plates.

FORCE EXERTED BY THE JET ON A STATIONARY (FIXED) VERTICAL PLATE

Let us consider a jet of water coming out from the nozzle, strikes a flat fixed vertical plate as shown in the figure.



Let $v =$ Velocity of the jet .

$d =$ Diameter of the jet .

$a =$ Area of cross-section of the jet $= \frac{\pi d^2}{4}$

$Q =$ Quantity of fluid striking the plate

$$= av = \frac{\pi d^2}{4} v .$$

We know that Density $\rho = \frac{\text{mass of the fluid}}{\text{Volume of the fluid}}$

or, mass of the fluid $= \rho \times$ Volume of the fluid .

or, mass of fluid issued by the jet per sec $= \frac{\text{mass}}{\text{sec}} = \frac{\rho \times \text{Volume}}{\text{sec}}$

The jet after striking the plate, will move

along the plate. But the plate is at right

angles to the jet . Hence the jet after

striking will get deflected through 90° .

Hence the component of the velocity of

jet in the direction of jet, after striking

will be zero

The force exerted by the jet on the plate
in the direction of jet .

$F_x =$ Rate of change of momentum in the
direction of force .

$$= \frac{\text{initial momentum} - \text{Final momentum}}{\text{Time}} .$$

$$F_x = \frac{\text{mass} \times \text{initial velocity} - \text{mass} \times \text{Final velocity}}{\text{Time}}$$

$$= \frac{\text{mass}}{\text{Time}} (\text{initial velocity} - \text{Final velocity})$$

$$= \frac{\text{mass}}{\text{sec}} [\text{velocity of jet before striking} - \text{velocity of jet after striking}]$$

$$= f a v [v - 0] = f a v^2$$

$$\boxed{F_x = f a v^2} \quad \text{or,} \quad \boxed{F_x = \frac{w}{g} a v^2 \text{ KN}} \quad \begin{matrix} \text{since } w = f \gamma \\ f = \frac{w}{g} \end{matrix}$$

In this case the work done by the jet on the plate is zero, since the plate is fixed i.e. stationary.

Problem: A jet of water of 100mm diameter impinges normally on a fixed plate with a velocity of 30 m/sec. Find the force exerted on the plate.

Solution: Data given

$$d = \text{Diameter of jet of water} = 100 \text{ mm} \\ = \frac{100}{1000} = 0.1 \text{ m}$$

$$v = \text{velocity of water} = 30 \text{ m/sec}$$

$$a = \text{Area of jet} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.1)^2 = 0.007857 \text{ m}^2$$

Force exerted on the plate

$$F_x = \rho a v^2 = 1000 \times 0.007857 \times (30)^2$$
$$= ~~7007~~ 7071.300 \text{ N}$$
$$= 7.07 \text{ kN.}$$

We can use another formula.

$$F_x = \frac{W}{g} a v^2 = \frac{9.81 \times 0.007857}{9.81} \times (30)^2$$
$$= 7.07 \text{ kN.}$$

Problem: - A jet of water, 75mm in diameter, issues with a velocity of 30m/sec and impinges on a stationary flat plate which destroys its forward motion. Find the force exerted by the jet on the plate and work done.

Solution: - Data given

$$d = \text{Diameter of the jet} = 75 \text{ mm}$$
$$= \frac{75}{1000} = 0.075 \text{ m}$$

$$v = \text{Velocity of jet} = 30 \text{ m/sec.}$$

$$a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (0.075)^2 = 0.004419 \text{ m}^2$$

The force exerted by the jet on a stationary vertical plate is given by

$$F_x = \rho a v^2$$

Putting all the values, we have

$$\begin{aligned} F_x &= 1000 \times 0.004419 \times (30)^2 \\ &= 3977.10 \text{ N.} \end{aligned}$$

As the plate is fixed, the work done is zero.

Problem :- A jet of water 50mm diameter is discharging under a constant head of 70 metres. Find the force exerted by the jet on a fixed plate. Take coefficient of velocity as 0.9

Solution :- Data given

$$d = \text{Diameter of jet} = 50\text{mm} = \frac{50}{1000} = 0.050 \text{ m}$$

$$H = \text{Constant Head} = 70\text{m.}$$

$$C_v = \text{Coefficient of velocity} = 0.9$$

$$\begin{aligned} \text{We know that } a &= \text{cross sectional area} \\ \text{of the jet} &= \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.05)^2 = 0.001964 \text{ m}^2 \end{aligned}$$

$$V = \text{Velocity of jet} = C_v \sqrt{2gH}$$

$$\begin{aligned} &= 0.9 \times \sqrt{2 \times 9.81 \times 70} = 0.9 \times \sqrt{1373.4} \\ &= 0.9 \times 37.0594 = 33.35 \text{ m/s} \end{aligned}$$

Force exerted by the jet on the fixed plate

$$F_x = \frac{\rho a v^2}{g} = \frac{9.81 \times 0.001964 \times (33.35)^2}{9.81}$$

$$= 2.18 \text{ kN (Ans)}$$

FORCE EXERTED BY A JET ON MOVING PARTS

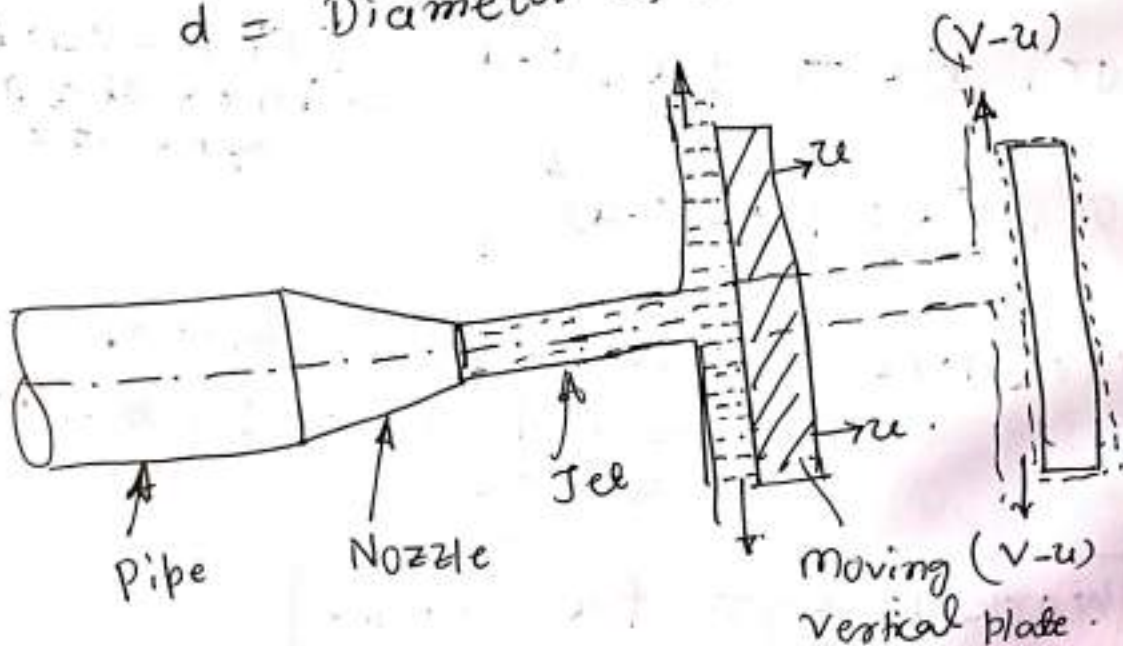
A jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.

Let v = Absolute velocity of the jet

a = Cross-sectional area of the jet

u = Velocity of plate held normal to the jet

d = Diameter of the jet



The relative velocity with which the jet strikes the plate is $(v-u)$

Mass of water striking the plate per second

$$= \rho \times \text{Area of jet} \times \text{velocity with which jet strikes the plate}$$

$$= \rho a (v-u)$$

Force exerted by the jet on the moving plate in the direction of the jet

$$\left[\begin{array}{l} \because \rho = \frac{\text{mass}}{\text{volume}} \\ \text{mass} = \rho \times \text{Volume} \\ = \rho a d \\ \Rightarrow \frac{\text{mass}}{\text{sec}} = \frac{\rho a d}{\text{sec}} = \rho a (v-u) \end{array} \right]$$

$$F_x = \text{mass of water striking the plate} \times \left(\frac{\text{initial}}{\text{sec}} \right)$$

velocity with which water strikes - Final velocity

$$= \rho a (v-u) \left[\frac{(v-u) - 0}{2} \right]$$

$$\text{or, } F_x = \rho a (v-u) \left[\begin{array}{l} \because \text{Final velocity} \\ \text{in the direction} \\ \text{of the jet} = 0 \end{array} \right]$$

$$\text{or, } F_x = \rho a (v-u)^2$$

Work done = Force \times Distance through which the body moves in the direction of force.

$$\text{Work done} = \rho a (v-u)^2 \times u$$

output of the jet per sec is the work done by the jet on the plate per sec $W = F_x u$

input of the jet per sec is the kinetic energy of the issuing jet per sec = $\frac{1}{2}(\rho a v) v^2$
 $= \frac{1}{2} \rho a v^3$

$$\eta_{jet} = \frac{F \cdot u}{\frac{1}{2} \rho a v^3} = \frac{\rho a (v-u) u}{\frac{1}{2} \rho a v^3}$$

Problem:~ A nozzle of 60 mm diameter delivers a stream of water at 24 m/sec perpendicular to a plate that moves away from the jet at 6 m/sec. Find (i) The force on the plate.

- (ii) The work done and
 (iii) The efficiency of the jet.

Solution:~ Data given

$d =$ Diameter of the jet = 60 mm = $\frac{60}{1000} = 0.06$ m

$V =$ Velocity of water = 24 m/sec

$u =$ Velocity of the plate = 6 m/sec.

$a =$ cross-sectional area = $\frac{\pi}{4} d^2 = \frac{\pi}{4} (0.06)^2 = 0.002828$ m²

(i) The force on the plate = $F = \rho a (v-u)^2$
 $= 1000 \times 0.002828 (24-6)^2 = 916$ N

(ii) Work done = $F \times u = 916 \times 6 = 5496$ J/sec

(iii) The efficiency of jet, η_{jet} .

Kinetic energy of issuing jet = $\frac{1}{2} m v^2$
 $= \frac{1}{2} (\rho a v) \cdot v^2$ [∵ $m = \rho a v$]

$$= \frac{1}{2} (1000 \times 0.002828 \times 24)(24)^2 = 19543.2 \frac{\text{Nm}}{\text{Sec}}$$

$$\eta_{\text{jet}} = \frac{\text{Work Done}}{\text{Kinetic energy of issuing jet}}$$

$$= \frac{5496}{19543.2} = 0.281 \text{ or } 28.1\%$$

Problem :- A jet of water of diameter 10cm strikes a flat plate normally with a velocity of 15 m/sec. The plate is moving with a velocity of 6 m/sec in the direction of the jet and away from the jet. Find

- (i) The force exerted by the jet on the plate.
 (ii) work done by the jet on the plate per second.

Solution :- Data given

$$d = \text{Diameter of the jet} = 10\text{cm} = \frac{10}{100} = 0.1\text{m}$$

$$a = \text{Area of the jet} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.1)^2 = 0.007854 \text{ m}^2$$

$$V = \text{Velocity of jet} = 15 \text{ m/sec}$$

$$u = \text{Velocity of the plate} = 6 \text{ m/sec.}$$

(i) The force exerted by the jet on a moving flat vertical plate is given

$$\text{by } F_x = \rho a (V-u)^2$$

$$= 1000 \times 0.007854 (15-6)^2$$

$$= 1000 \times 0.007854 \times (9)^2 = 636.17 \text{ N.}$$

(ii) Work Done per second by the jet

$$= F \times U = 636.17 \times 6 = 3817.02 \text{ Nm/sec.}$$

Problem :- A jet of water 50mm diameter and moving with a velocity of 26m/sec is impinging normally on a plate.

Determine the pressure on the plate, when (a) it is fixed and (b) it is moving with a velocity of 10m/sec in the direction of the jet. Also determine the work done per second by the jet.

Solution :- Data given

$$d = 50\text{mm} = \frac{50}{1000} = 0.050 \text{ m.}$$

$$V = 26\text{m/sec}, \quad U = 10\text{m/sec.}$$

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.05)^2 = 0.001964 \text{ m}^2$$

(a) pressure on the plate when it is fixed

$$P_1 = \rho a V^2 = 1000 \times 0.001964 \times (26)^2 = 1327.67 \text{ N.}$$

(b) pressure on the plate when it is moving

$$= P_2 = \rho a (V - U)^2$$

$$= 1000 \times 0.001964 (26 - 10)^2 = 502.78 \text{ N}$$

(c) work done by the jet = Force \times Distance.

$$= 502.78 \times 10 = 5027.80 \text{ Joule.}$$

DERIVATION OF WORK DONE ON SERIES OF VANES AND CONDITION FOR MAXIMUM EFFICIENCY

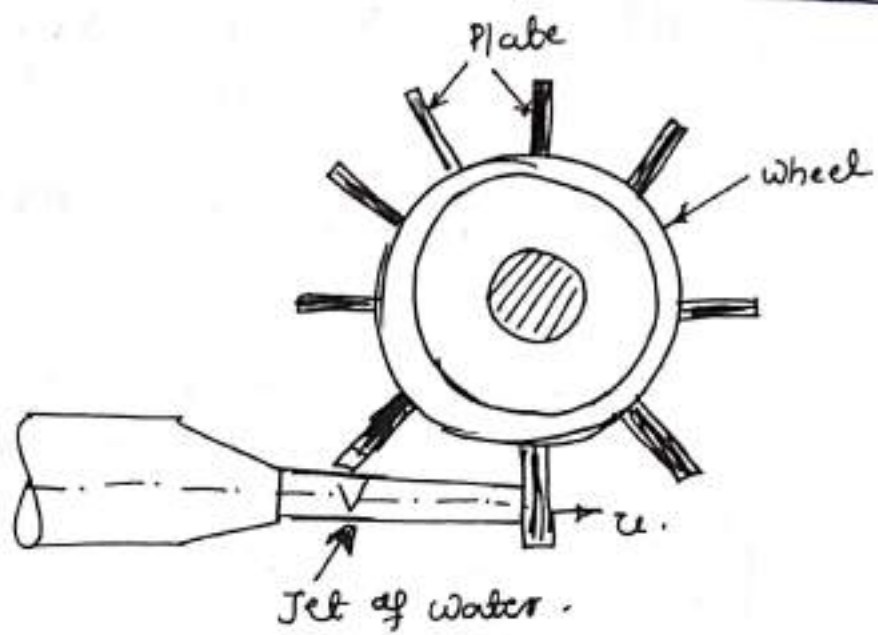
A large number of plates are mounted on the circumference of a wheel at a fixed distance apart as shown in the figure. The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving and the second plate mounted on the wheel appears before the jet, which again exerts the force on the second plate. Thus each plate appears successively before the jet and the jet exerts force on each plate. The wheel starts moving at a constant speed.

Let $V =$ velocity of jet

$d =$ Diameter of jet

$a =$ Cross-sectional area of jet
 $= \frac{\pi}{4} d^2$

$v =$ velocity of vane



Jet striking a series of vanes.

In this case the mass of water coming out from the nozzle per second is always in contact with the plates, when all the plates are considered

Hence mass of water per second striking the series of plates = $\rho a v$.

The jet strikes the plate with a velocity $(v-u)$

After striking the jet moves tangential to the plate and hence the velocity component in the direction of motion of plate is equal to zero

The force exerted by the jet in the direction of motion of plate.

$$F_x = \text{mass per second} [\text{initial velocity} - \text{Final velocity}]$$

$$= \rho a v [(v-u) - 0] = \rho a v [v-u].$$

$$u = \frac{\pi DN}{60}, \text{ where } D = \text{wheel diameter}$$

$$N = \text{speed of wheel}$$

Work done by the jet on the series of plates per second

$W = \text{Force} \times \text{Distance per second in the direction of force.}$

$$W = F \times u$$

$$W = \rho a v [v-u] \times u$$

Kinetic energy of the jet per second

$$= \frac{1}{2} m v^2 = \frac{1}{2} (\rho a v) \times v^2 = \frac{1}{2} \rho a v^3$$

$$\text{Efficiency } = \eta = \frac{\text{Work done per second}}{\text{Kinetic energy per second}}$$

$$= \frac{\rho a v (v-u) \times u}{\frac{1}{2} \rho a v^3} = \frac{(v-u)u}{\frac{1}{2} v^2}$$

$$\text{Efficiency} = \frac{2u[v-u]}{v^2}$$

CONDITION FOR MAXIMUM EFFICIENCY :-

$$\text{The equation efficiency} = \frac{2u[v-u]}{v^2}$$

gives the value of the efficiency of the wheel. For a given jet velocity v , the efficiency will be maximum, when

$$\frac{d\eta}{du} = 0 \quad \text{or, } \frac{d}{du} \left[\frac{2u[v-u]}{v^2} \right] = 0$$

$$\text{or, } \frac{d}{du} \left[\frac{2uv - 2u^2}{v^2} \right] = 0$$

$$\text{or, } \frac{1}{v^2} \left[\frac{d}{du} [2uv - 2u^2] \right] = 0$$

$$\text{or, } \frac{1}{v^2} [2v - 4u] = 0, \quad 2v - 4u = 0 \times v^2 = 0$$

$$\text{or, } 2v - 4u = 0 \quad \text{or, } 2v = 4u \therefore v = \frac{4u}{2}$$

$$\text{or, } v = 2u \therefore \boxed{u = \frac{v}{2}}$$

MAXIMUM EFFICIENCY: ~ Putting the value
 $v = 2u$ in equation $\eta = \frac{2u[v-u]}{v^2}$, we get

maximum efficiency as

$$\eta_{\text{max}} = \frac{2u[2u-u]}{(2u)^2} = \frac{2u[u]}{4u^2} = \frac{1}{2} \text{ or } 50\%$$

CHAPTER : 6.0 FLOW THROUGH PIPES.

INTRODUCTION:

A pipe is a closed conduit. It is circular cross-sectional section and used to carry water or any other fluid. The flow in a pipe is termed pipe flow only when the fluid completely fills the cross section and there is no free surface of fluid. When the pipe is running full, the flow is under pressure, but if the pipe is not running full, the flow is not under pressure. It is seen in the case of sewer pipes, culverts etc.

[Sewer pipes: It means channel for carrying water and refuse from drain].

Generally the fluid flowing in a pipe is always subjected to resistance due to shear forces between fluid particles and the boundary walls of the pipe and between the fluid particles themselves. The resistance to the flow of fluid is in general known as frictional

resistance. In generally the flow of fluid in a pipe may be either laminar or turbulent.

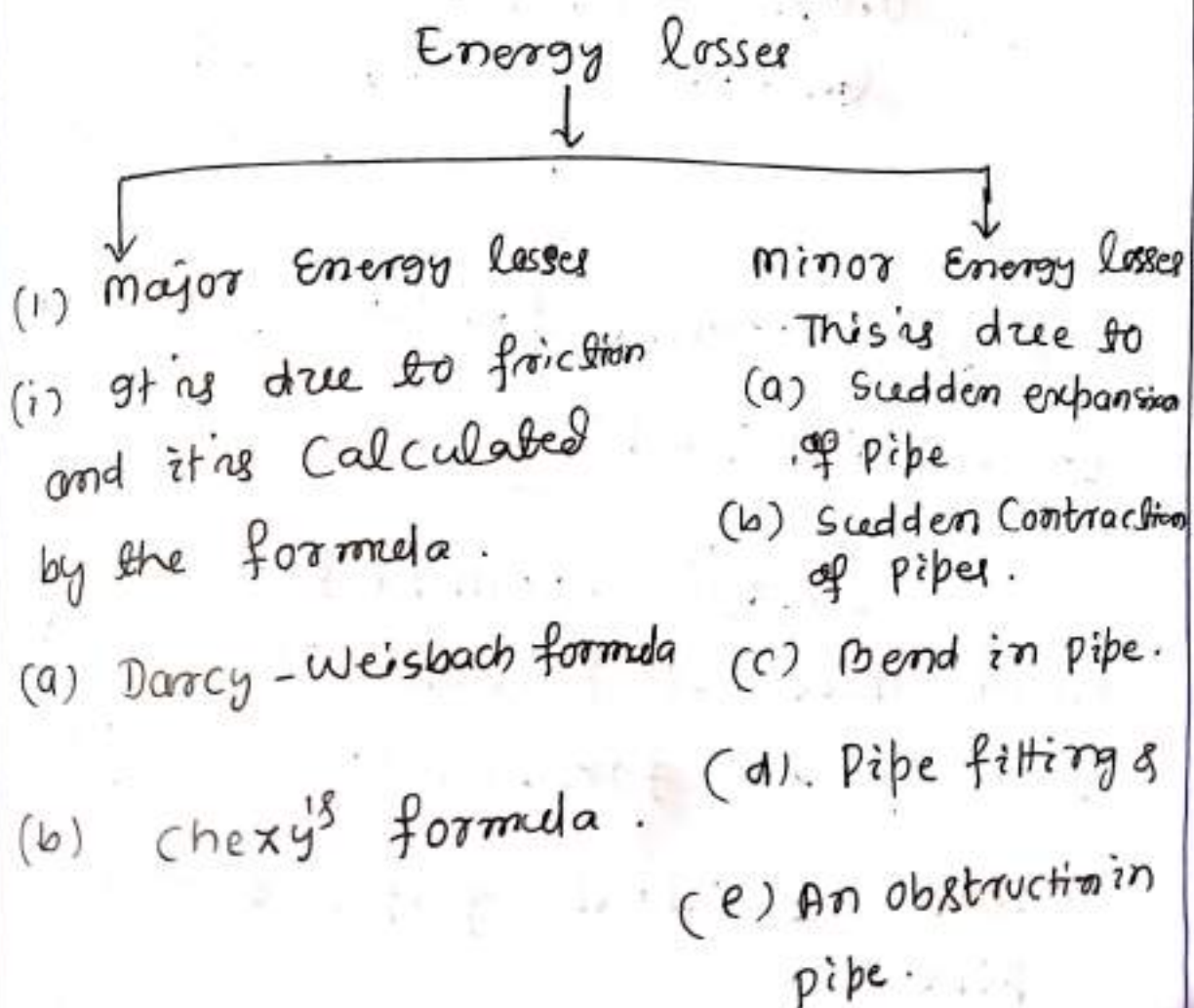
LOSS OF HEAD (ENERGY) IN PIPES :-

When water flows in a pipe, it creates some resistance to its motion, whose effect is to reduce the velocity and ultimately the head of water available. Actually there are many types of losses, but the major losses is due to frictional resistance of a pipe depends upon the roughness of the inside of the pipe. We know from experimentally that more of the roughness of the inside surface of the pipe. We know from experimentally that more the roughness of the inside surface of the pipe, greater will be the resistance. This friction

is known as fluid resistance and the resistance is known as frictional resistance.

LOSS OF ENERGY IN PIPES.

When water flows in a pipe, it experiences some resistance to its motion, due to which its velocity and ultimately the head of water available is reduced. This loss of energy (or head) is classified as follows.



LOSS OF ENERGY (HEAD) DUE TO FRICTION

The loss of head (Energy) in pipes due to friction is calculated from Darcy Weisbach equation and is given by

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \cdot 2g}$$

where h_f = Loss of head due to friction.

f = Co-efficient of friction.

It is a function of Reynolds number
 $= \frac{16}{Re}$ for $Re < 2000$ [Laminar flow]

$$f = \frac{0.0791}{(Re)^{1/4}}$$

where L = Length of the pipe.
 V = mean velocity of flow
 d = Diameter of the pipe.

Chezy's Formula for Loss of Head due to friction in pipes.

Chezy's formula for loss of head due to friction in pipes is given by formula $V = C \sqrt{mi}$

where V = Velocity of flow through pipe.

C is a constant known as Chezy's constant

$$\text{and } \frac{h_f}{L} = i$$

i is loss of head ~~due to~~ per unit length of pipe.

$$m = \text{Hydraulic mean depth} = \frac{A}{P} = \frac{\frac{\pi d^2}{4}}{\pi d} = \frac{d}{4}$$

Problem: ~ Find the loss of head, due to friction in a pipe of 1 metre diameter and 15 km long. The velocity of water in the pipe is 1 metre/sec. Take coefficient of friction is 0.005.

Solution: ~ Data given.

d = Diameter of the pipe = 1 metre.

L = Length of the pipe = 15 km = 15000 m

v = velocity of water = 1 m/sec.

f = Co-efficient of friction = 0.005

h_f = Loss of head due to friction.

$$h_f = \frac{4 \cdot f \cdot L \cdot v^2}{d \cdot 2g} = \frac{4 \times 0.005 \times 15000 \times (1)^2}{1 \times 2 \times 9.81}$$

$$h_f = 15.29 \text{ m}$$

Problem 2 :- In a pipe of diameter 350mm and length 75m water is flowing at a velocity of 2.8 m/sec. Find the head lost due to friction using (i) Darcy-Weisbach formula

(ii) Chezy's formula for which $C = 55$

Assume kinematic viscosity of water as 0.012 stoke.

Solution :- Data given

$$d = \text{Diameter of the pipe} = 350 \text{ mm} \\ = \frac{350}{1000} = 0.350 \text{ m.}$$

$$L = \text{Length of the pipe} = 75 \text{ m.}$$

$$V = \text{Velocity of the fluid} = 2.8 \text{ m/sec.}$$

$$C = \text{Chezy's Constant} = 55$$

$$\nu = \text{Kinematic viscosity of water} \\ = 0.012 \text{ stoke} = 0.012 \text{ cm}^2/\text{sec} \\ = 0.012 \times 10^{-4} \text{ m}^2/\text{sec.}$$

(i) Darcy-Weisbach formula is given by

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \cdot 2g}$$

where f = Co-efficient of friction.

$$Re = \frac{V \cdot d}{\nu} = \frac{2.8 \times 0.35}{0.012 \times 10^{-4}} = 8.167 \times 10^5$$

$$f = \frac{0.0791}{(Re)^{1/4}} = \frac{0.0791}{(8.167 \times 10^5)^{1/4}} = 0.00263$$

Head Lost due to friction

$$h_f = \frac{4 \cdot f \cdot l \cdot v^2}{d \cdot 2g} = \frac{4 \times 0.00263 \times 75 \times (2.8)^2}{0.35 \times 2 \times 9.81}$$

$$h_f = 0.9 \text{ m}$$

(ii) Chezy's formula is given by $V = C \sqrt{mi}$
where $C = 55$, $m = \frac{A}{P} = \frac{\frac{\pi d^2}{4}}{\pi d} = \frac{d}{4} = \frac{0.35}{4}$
or, $m = 0.0875 \text{ m}$.

Putting all the values, we have

$$\Rightarrow 2.8 = 55 \sqrt{0.0875 \cdot i}$$

$$\text{or, } \frac{2.8}{55} = \sqrt{0.0875 \cdot i} \quad \text{or, } \left(\frac{2.8}{55}\right)^2 = 0.0875 \cdot i$$

$$\text{or, } 0.0025 = 0.0875 \cdot i \quad \therefore i = \frac{0.0025}{0.0875} = 0.0296$$

$$\text{but } i = \frac{h_f}{L} = 0.0296 \quad \text{or, } \frac{h_f}{75} = 0.0296$$

$$\therefore h_f = 0.0296 \times 75 = 2.22 \text{ m}$$

$$h_f = 2.22 \text{ m}$$

Problem: Find the diameter of a pipe of length 2000m when the rate of flow of water through the pipe is 200 litres/sec. and the head lost due to friction is 4m. Take the value of $C = 50$ in Chezy's formula.

Solution: Data given

$$l = \text{length of the pipe} = 2000 \text{ m}$$

$$Q = \text{Rate of flow} = 200 \text{ lit/sec} = \frac{200}{1000} \\ = 0.2 \text{ m}^3/\text{sec}$$

$$h_f = \text{Head lost due to friction} = 4 \text{ m.}$$

$$C = \text{Value of Chezy's Constant} = 50$$

d = Let the diameter of the pipe

$$\text{We know that } Q = A \cdot V \text{ or, } V = \frac{Q}{A} = \frac{0.2}{\frac{\pi}{4} d^2}$$

$$\text{or, } V = \frac{0.2 \times 4}{\pi d^2}$$

$$m = \text{Hydraulic mean depth} = \frac{d}{4}$$

i = Loss of head per unit length

$$i = \frac{h_f}{L} = \frac{4}{2000} = 0.002.$$

We know from Chezy's formula

$$V = C \sqrt{mi} \text{ or, } \frac{0.2 \times 4}{\pi d^2} = 50 \sqrt{\frac{d}{4} \times 0.002}$$

$$\text{or, } \frac{0.2 \times 4}{\pi \times 50 \times d^2} = \sqrt{\frac{d}{4} \times 0.002}$$

$$\text{or, } \frac{0.00509}{d^2} = \sqrt{\frac{d}{4} \times 0.002}$$

Squaring both sides, we get

$$\frac{d}{4} \times 0.002 = \left(\frac{0.00509}{d^2} \right)^2 = \frac{0.0000259}{d^4}$$

$$\text{or, } d^5 = \frac{0.000259 \times 4}{0.002} = 0.0518$$

$$\text{or, } d = \sqrt[5]{0.0518} = (0.0518)^{\frac{1}{5}} = 0.553 \text{ m}$$

$$d = 0.553 \times 1000 = 553 \text{ mm}$$

Problem: - Water is flowing through a pipe 1500m long with a velocity of 0.8m/sec. What should be the diameter of the pipe, if the loss of head due to friction is 8.7m. Take 'f' for the pipe as 0.01.

Solution: - Data given

l = length of the pipe = 1500m.

v = velocity of water = 0.8m/sec.

d = Diameter of the pipe = ?

h_f = Head lost due to friction = 8.7m

f = Co-efficient of friction = 0.01

We know that $h_f = \frac{4flv^2}{d \times 2g}$.

Putting all the values, we have

$$8.7 = \frac{4 \times 0.01 \times 1500 \times (0.8)^2}{d \times 2 \times 9.8}$$

$$\text{or, } 8.7 \times d \times 2 \times 9.8 = 4 \times 0.01 \times 1500 \times (0.8)^2$$

$$\text{or, } d = \frac{4 \times 0.01 \times 1500 \times (0.8)^2}{8.7 \times 2 \times 9.8} = 0.225 \text{ m} = 0.225 \times 1000$$

$$d = 225 \text{ mm}$$

Problem :- Find the loss of head, due to friction, in a pipe of 500mm diameter and 1.5 kilometres long. The velocity of water in the pipe is 1 m/sec. Take coefficient of friction = 0.005

Solution :- Data given.
 $d = \text{Diameter of the pipe} = 500 \text{ mm} = \frac{500}{1000}$
 $= 0.5 \text{ m}.$

$$f = 0.005$$

$$l = \text{Length of the pipe} = 1.5 \text{ km} = 1500 \text{ m}$$

$$V = \text{Velocity of water} = 1 \text{ m/sec}.$$

$$h_f = \text{Head loss due to friction} = ?$$

$$h_f = \frac{4flv^2}{d \times 2g} = \frac{4 \times 0.005 \times 1500 \times (1)^2}{0.5 \times 2 \times 9.8} = 3.06 \text{ m}$$

$$h_f = 3.06 \text{ m}$$

Problem :- In a pipe of 300mm diameter and 800m length an oil of specific gravity 0.8 is flowing at the rate of $0.45 \text{ m}^3/\text{sec}$. Find

(i) Head lost due to friction.

(ii) Power required to maintain the flow

Take kinematic viscosity of oil as $0.3 \text{ stoke}.$

Solution :- Data given.

$$d = \text{Diameter of the pipe} = 300 \text{ mm} = \frac{300}{1000} = 0.3 \text{ m}.$$

$$l = \text{Length of the pipe} = 800 \text{ m}.$$

$$s = \text{specific gravity of oil} = 0.8.$$

$$\nu = \text{Kinematic viscosity of oil} = 0.3 \text{ stoke}$$

$$= 0.3 \text{ cm}^2/\text{sec} = 0.3 \times 10^{-4} \text{ m}^2/\text{sec}.$$

$$Q = \text{Discharge} = 0.45 \text{ m}^3/\text{sec}.$$

(i) Head lost due to friction, $= h_f$.

We know that $Q = AV \therefore V = \frac{Q}{A}$.

$$\therefore V = \frac{Q}{\frac{\pi}{4} d^2} = \frac{0.45}{\frac{\pi}{4} \times (0.3)^2} = 6.363 \text{ m/sec}.$$

$$Re = \text{Reynold's number} = \frac{V \times D}{\nu} = \frac{6.363 \times 0.3}{0.3 \times 10^{-4}}$$
$$= 6.363 \times 10^4$$

$$f = \text{CO-efficient of friction} = \frac{0.0791}{(6.363)^{0.25}}$$

$$= \frac{0.0791}{(6.363 \times 10^4)^{0.25}} = 0.00498$$

$$h_f = \frac{4 f l v^2}{d \times 2g} = \frac{4 \times 0.00498 \times 800 \times (6.363)^2}{0.3 \times 2 \times 9.81}$$

$$h_f = 109.61 \text{ m}.$$

(ii) Power required, P

Power required to maintain the flow where $w = 0.8 \times 9.81 = 7.848 \text{ kN/m}^3$

$$P = w Q h_f$$

$$h_f = 109.61, Q = 0.45 \text{ m}^3/\text{sec}.$$

$$P = 7.848 \times 0.45 \times 109.61 = 387.09 \text{ kW}$$

$$\therefore P = 387.09 \text{ kW}$$

MINOR ENERGY (HEAD) LOSSES: ~ The

loss of Head or Energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the flowing fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases.

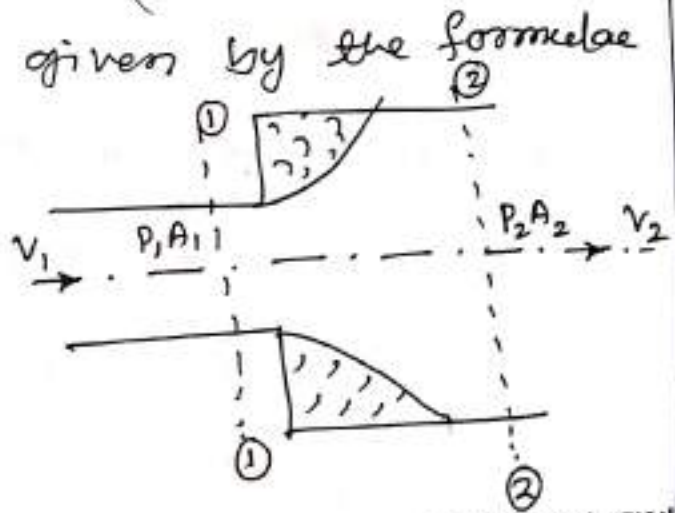
- (i) Loss of head due to sudden enlargement.
- (ii) Loss of head due to sudden contraction.
- (iii) Loss of head due to obstruction in the pipe.
- (iv) Loss of head at the entrance to a pipe.
- (v) Loss of head at the exit of a pipe.
- (vi) Loss of head due to bend in the pipe.
- (vii) Loss of head in various pipe fittings.

LOSS OF HEAD DUE TO SUDDEN ENLARGEMENT:

Let us consider liquid flowing through a pipe which has sudden enlargement. Due to sudden enlargement, the flow is decelerated abruptly and eddies are developed resulting in loss of energy (or head)

Loss of Energy (Head) due to sudden enlargement is given by the formulae

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

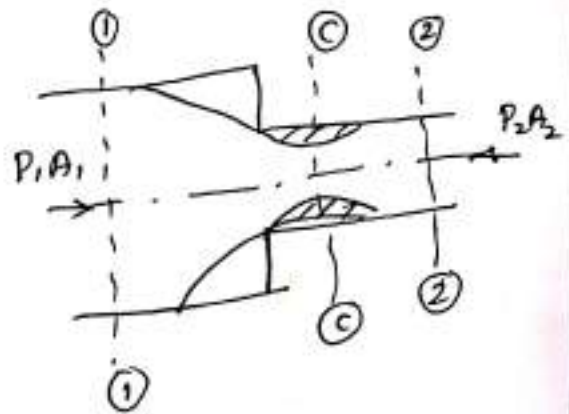


LOSS OF HEAD DUE TO SUDDEN CONTRACTION

Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in the figure. Let two sections 1-1, and 2-2 before and after contraction. As the liquid flows from larger pipe to smaller pipe, the area of flow goes on decreasing and becomes

minimum at a section c-c as shown in the figure. The section c-c is called Vena-Contracta.

After section c-c a sudden enlargement of the area takes place.



The loss of head due to sudden contraction is actually due to sudden enlargement from Vena-Contracta to smaller pipe.

It is given by the formulae

$$h_c = \frac{K V_2^2}{2g} = 0.375 \frac{V_2^2}{2g} \quad \because C_c = \frac{A_c}{A_2}$$

$$K = \left[\frac{1}{C_c} - 1 \right]^2$$

If the value of C_c is not given then

the head loss due to contraction is

taken as -
$$h_c = \frac{0.5 V_2^2}{2g}$$

If the value of C_c is assumed to

be equal to 0.62.

Then
$$K = \left[\frac{1}{0.62} - 1 \right]^2 = 0.375$$

Problem: Find the loss of head when a pipe of diameter 200mm is suddenly enlarged to a diameter of 400mm. The rate of flow of water through the pipe is 250 lit/sec.

Solution: Data given

$$D_1 = \text{Diameter of smaller pipe} = 200\text{mm} = \frac{200}{1000} = 0.20\text{m}$$

$$\text{Area} = A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314\text{m}^2$$

$$D_2 = \text{Diameter of larger pipe} = 400\text{mm} = \frac{400}{1000} = 0.40\text{m}$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.40)^2 = 0.1257\text{m}^2$$

$$Q = \text{Discharge through the pipe} = 250 \text{ lit/sec} \\ = \frac{250}{1000} = 0.25 \text{ m}^3/\text{sec} \quad \left[\because 1 \text{ lit} = \frac{1000}{1000} \text{ cm}^3 = 1 \text{ m}^3 \right]$$

We know that $Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.25}{0.0314} = 7.96 \text{ m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.1257} = 1.99 \text{ m/sec}$$

Loss of head due to sudden enlargement is given by $h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2 \times 9.81}$

$$h_e = \frac{(5.97)^2}{2 \times 9.81} = 1.816 \text{ m of water}$$

Problem :- The rate of flow of water through a horizontal pipe is $0.25 \frac{m^3}{sec}$. The diameter of the pipe which is $200mm$ is suddenly enlarged to $400mm$. The pressure intensity in the smaller pipe is $11.772 \frac{N}{cm^2}$. Determine (i) Loss of head due to sudden enlargement, (ii) pressure intensity in the larger pipe (iii) Power lost due to enlargement.

Solution :- Data given $Q = 0.25 \frac{m^3}{sec}$
 $D_1 =$ Diameter of the smaller pipe = $200mm$
 $= \frac{200}{1000} = 0.20m$.
 Area of the smaller pipe $= \frac{\pi}{4} \times (D_1)^2 = \frac{\pi}{4} \times (0.20)^2$
 $= 0.03141m^2$.

$D_2 =$ Diameter of the larger pipe = $400mm$
 $= \frac{400}{1000} = 0.40m$.

Area of the larger pipe $= \frac{\pi}{4} \times (D_2)^2$
 $= \frac{\pi}{4} \times (0.40)^2 = 0.1257m^2$

Pressure in smaller pipe $= 11.772 \frac{N}{cm^2}$
 $= 11.772 \frac{N}{10^4 m^2} = 11.772 \times 10^4 \frac{N}{m^2}$

Now velocity in smaller pipe = $v_1 = \frac{Q}{A_1}$

$$\text{or, } v_1 = \frac{0.25}{0.0314} = 7.96 \text{ m/sec}$$

Velocity in larger pipe $v_2 = \frac{Q}{A_2} = \frac{0.25}{0.1257} = 1.99 \text{ m/sec}$

(i) Loss of head due to sudden enlargement

$$h_e = \frac{(v_1 - v_2)^2}{2g} \quad \text{putting all the values,}$$

we get $h_e = \frac{(7.96 - 1.99)^2}{2 \times 9.81} = \frac{(5.97)^2}{2 \times 9.81}$

$$= \frac{35.6409}{2 \times 9.81} = 1.816 \text{ m.}$$

$$h_e = 1.816 \text{ m}$$

(ii) Let P_2 = pressure intensity in larger pipe
Then applying Bernoulli's equation before
and after the sudden enlargement.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_e$$

Since the pipe is horizontal $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_e$$

$$\text{or, } \frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} - h_e$$

$$= \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{(7.96)^2}{2 \times 9.81} - \frac{(1.99)^2}{2 \times 9.81} - 1.816$$

$$= 12 + 3.229 - 0.208 - 1.816$$

$$\text{or, } \frac{P_2}{\rho g} = 15.229 - 2.0178 = 13.21$$

$$\text{or, } P_2 = 13.21 \times \rho g = 13.21 \times 1000 \times 9.81$$

$$\text{or, } \boxed{P_2 = 12.96 \text{ N/cm}^2}$$

(iii) Power lost due to Sudden enlargement

$$P = \frac{\rho g Q h_e}{1000} = \frac{1000 \times 9.81 \times 0.25 \times 1.816}{1000}$$

$$\therefore \boxed{P = 4.453 \text{ Kw}}$$

Problem :- Calculate the discharge through a pipe of diameter 200mm when the difference of pressure head between the two ends of a pipe 500m apart is 4m of water. Take the value of 'f' = 0.009 in the

formula
$$h_f = \frac{4f l v^2}{d \times 2g}$$

Solution :- Data given

$$d \text{ } \textcircled{\text{D}} = \text{Diameter of the Pipe} = 200\text{mm} = \frac{200}{1000} \\ = 0.20\text{m}.$$

$$l = \text{Length of the Pipe} = 500\text{m}.$$

$$h_f = \text{Difference of pressure head} = 4\text{m of water}.$$

$$f = 0.009$$

We know that $h_f = \frac{4 \cdot f \cdot l \cdot v^2}{d \cdot 2g}$

$$\text{or, } 4.0 = \frac{4 \times 0.009 \times 500 \times v^2}{0.2 \times 2 \times 9.81} = 4.587 v^2$$

$$\text{or, } v^2 = \frac{4.0}{4.587} = 0.872 \quad \text{or, } v = \sqrt{0.872}$$

$$\text{or, } v = 0.933 \text{ m/sec}$$

\therefore Discharge $Q = \text{Area} \times \text{Velocity}$

$$\text{or, } Q = \frac{\pi}{4} d^2 \times 0.933 = \frac{\pi}{4} \times (0.20)^2 \times 0.933$$

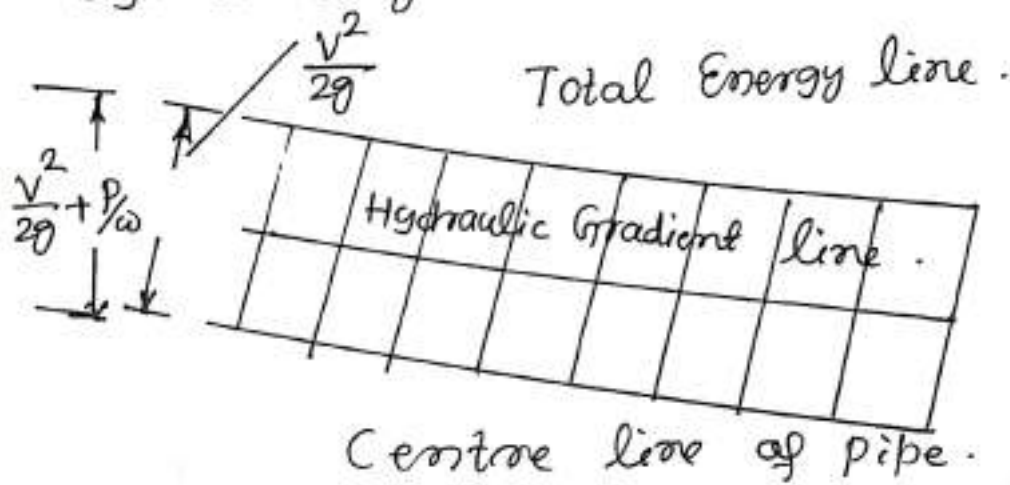
$$\text{or, } Q = 0.0293 \text{ m}^3/\text{sec} = 0.0293 \times 1000 = 29.3 \frac{\text{lit}}{\text{sec}}$$

$$\therefore \boxed{Q = 29.3 \frac{\text{lit}}{\text{sec}}}$$

EXPLAIN TOTAL ENERGY LINE AND
HYDRAULIC GRADIENT LINE.

Hydraulic Gradient line is an imaginary line which indicates the pressure head at different section conveying fluid in between two tanks. The line shows the piezometric head at different section of the pipe line. It shows piezometric heads or pressure heads ($P/\rho g$) of a liquid flowing

in a pipe, be plotted as vertical ordinates on the centre line of the pipe, then the line joining the tops of such ordinates is known as hydraulic gradient line. It is symbolically written as (H.G.L)



TOTAL ENERGY LINE :- ~ of the sum

of pressure head and velocity heads $(\frac{P}{\omega} + \frac{v^2}{2g})$ of a liquid flowing in a pipe,

be plotted as vertical ordinates on the centre line of the pipe, then the line joining the tops of such ordinates is known as total energy line. It is symbolically written as (T.E.L)

In other words, the total Energy line lies, over the hydraulic gradient, by an amount equal to the velocity heads as shown in the diagram.