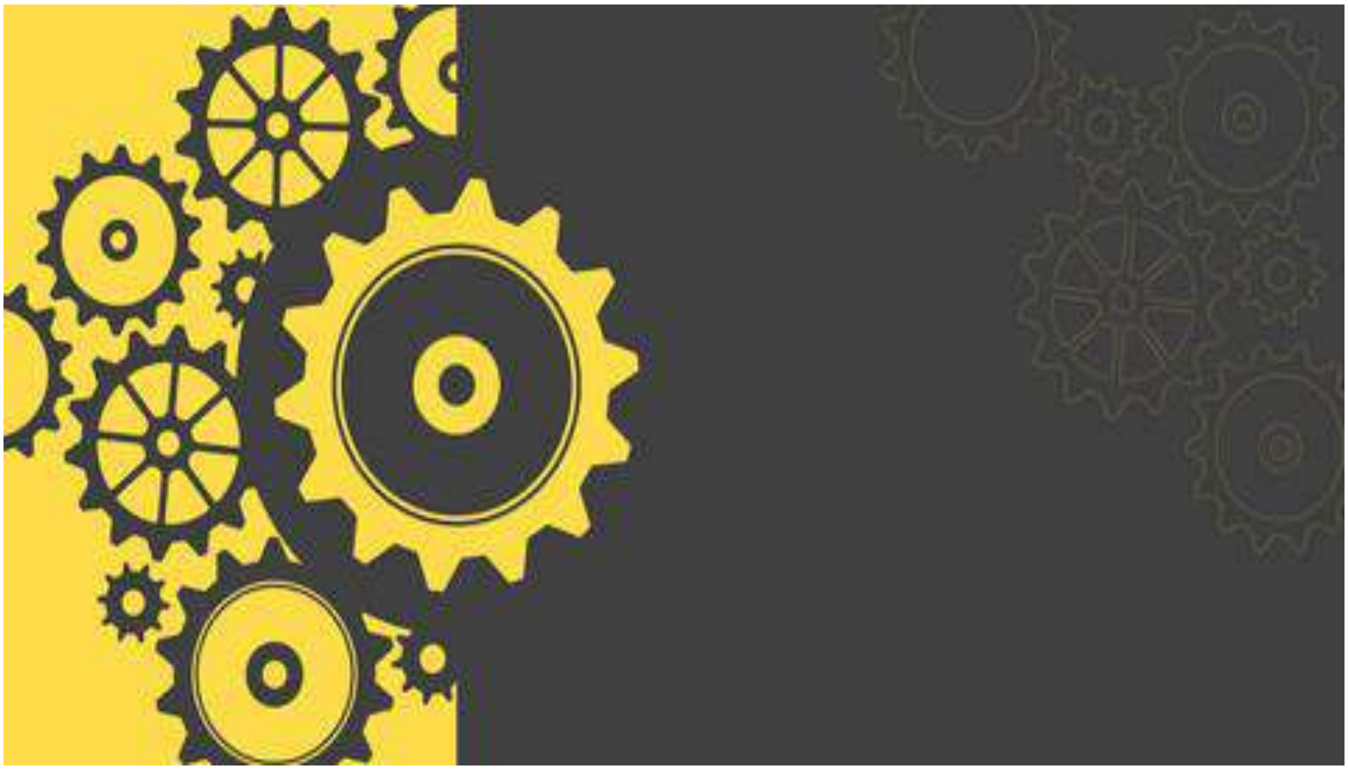
Engineering Mechanics

Of

$1^{st}/2^{nd}$ semester of all Engineering Branches



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Fundamental of Engineering Mechanics

Engineering mechanics :

* The subject Engg. mechanics is that branch of applied science, which deals laws and principles of. mechanics and their application to Engg. problem.

* Engg. mechanics is the study of force and it's effect on the body.

Classification of Engg. Mechanics :

The subject Engg. mechanics may divided into two main groups.

(1) Statics :

It is the branch of Engg. mechanics, which deals with the force and their effect while acting upon bodies at rest.

(2) Dynamics :

It is that branch of Engg. mechanics, which deals with the forces and their effect while acting upon the bodies in motion.

Dynamics further divided into two branches.

 # Kinetics : It is the branch of dynamics which deals with the bodies in motion due to the application of forces.

 # Kinematics : It deals with the bodies in motion without any reference to the force which are responsible for motion.

Rigid body:

A body is said to be rigid if it does not undergo deformation, whatever force may be applied to the body.

Elastic body:

A body is said to be elastic if it completely regains it's original shape after the removal of force.

Force:

Force is an external agent capable of changing a body's state of rest or motion.

* It has a magnitude and a direction.

* The SI unit of force is newton (N).

mathematically,

$$\text{Force} = \text{mass} \times \text{Acceleration}$$

$$\Rightarrow F = ma$$

Effect of force:

(i) Force changes the motion of a body.

(ii) It may increase the internal stresses in the body.

(iii) Force can change the direction of moving object.

(iv) Force can change the shape and size of an object.

Characterstic of a force :

(i) magnitude of force (i.e 50N, 60N, 100 N etc)

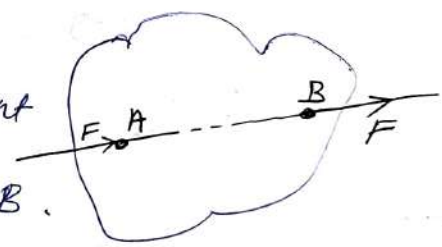
(ii) Direction of force (i.e West, 30° North of south etc.)

(iii) Nature of the force (i.e push or pull)

(iv) Principle of transmissibility of force

It States "If a force act at any point on a rigid body. It may be considered to act at any point on it's line of action".

Here, magnitude of force at point A = magnitude of force at point B.

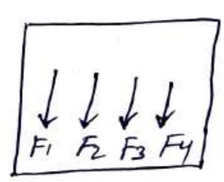


System of force :

When two or more force act on a body is called System of force. Force system is basically classified into following types.

① **Coplanar forces** : The forces, whose line of action lie on the same plane called Coplanar force.

(collinear forces)

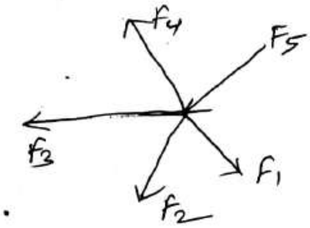


(Coplanar forces)

② **collinear forces** :

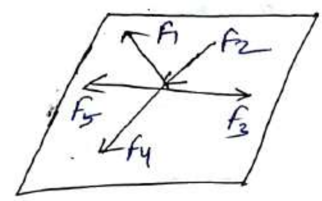
The forces, whose line of action lies on the same line are called collinear forces.

③ concurrent forces : The forces whose line of action pass through a common point are called concurrent force.



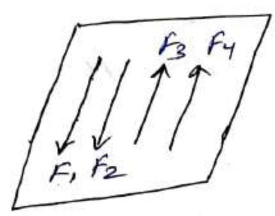
④ Coplanar concurrent forces :

The forces whose line of action lie in the same plane and at the same time pass through a common point are called coplaner concurrent forces



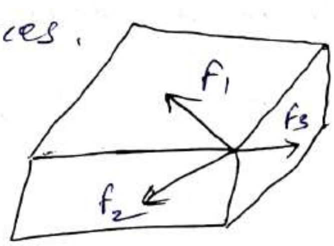
⑤ Coplanar Non-Concurrent forces :

The forces which do not meet at one point but their line of action lie on the same plane are called coplaner non-concurrent force.



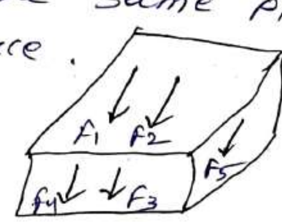
⑥ Non-Coplanar Concurrent force :

The forces which meet at one point but their lie of action do not lie on the same plane are called non-coplanar concurrent forces.



⑦ Non-coplanar non-concurrent force :

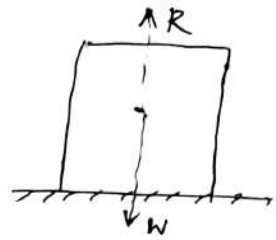
The forces which do not meet at one point & their line of action do not lie on the same plane called non-coplanar non-concurrent force.



Action & Reaction :

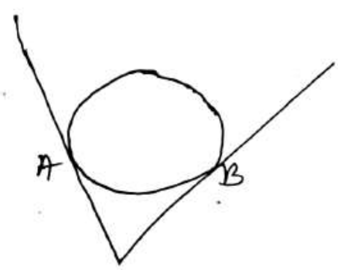
* Action means active force, when a body having a weight ($W = mg$) is placed on a horizontal surface, the body exerts vertically downward force equal to 'W'. It is called action.

* Reaction means reactive force. According to the newtons 3rd law of motion, "Every action has an equal and opposite reaction". So the horizontal surface exert reaction force 'R' vertically upward direction.

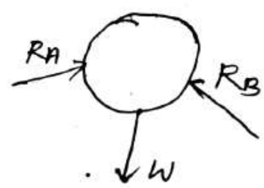


Free body diagram :

The representation of reaction force on the body by removing all the support or force act from the body called free body diagram



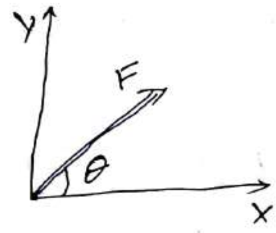
(Object with support)



(free body diagram)

Resolution of a force

The process of splitting up the given force into number of components, without changing it's effects on the body called resolution of a force.

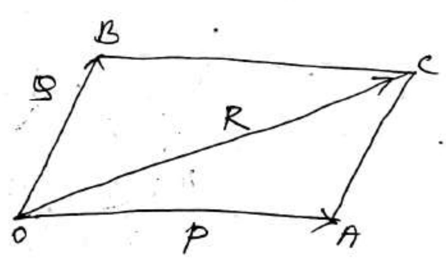


By resolution of force F we found,

$$X = F\cos\theta, \quad Y = F\sin\theta$$

Resultant of forces :

Resultant of two or more forces is a single forces whose effect on a body is the same as the given forces taken together acting on the body.



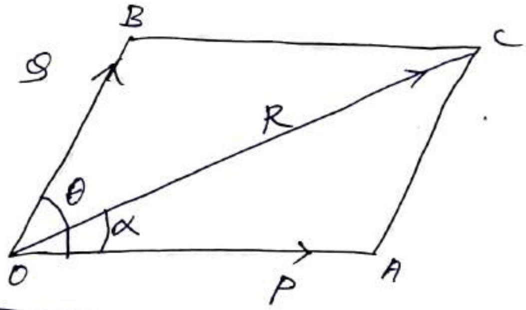
Here, R is the resultant of two forces P and Q.

Method for finding the resultant force :

1. Analytical method
   - (a) parallelogram law of force
   - (b) method of resolution
2. Graphical method.
   - (a) Triangle law of force.
   - (b) polygon law of force.

(a) parallelogram law of force :

This theorem states that " If two forces acting at a point be represented in magnitude & direction by the two adjacent sides of a parallelogram drawn from a point, then their resultant is represented in magnitude & direction by the diagonal of the parallelogram.



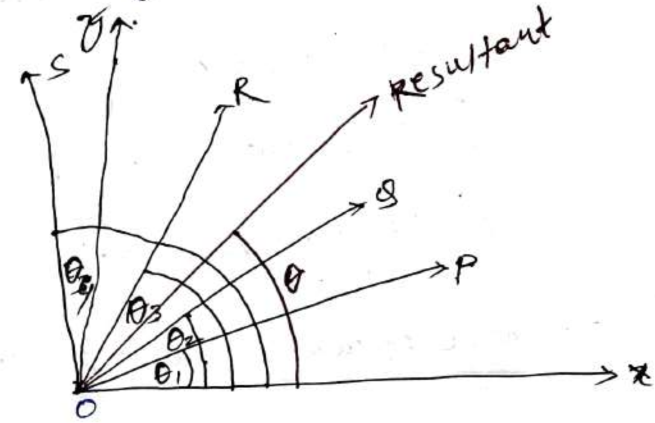
Resultant force, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Direction of resultant force.

$$\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

(b) method of resolution

When number of coplaner concurrent force acting on the body. we use method of resolution of force for finding resultant and direction of forces.



Resolve force in horizontal direction (i.e OX)

$$\Sigma H = P\cos\theta_1 + Q\cos\theta_2 + R\cos\theta_3 + S\cos\theta_4$$

Similarly, resolve force in vertically (i.e OX)

$$\Sigma V = P\sin\theta_1 + Q\sin\theta_2 + R\sin\theta_3 + S\sin\theta_4$$

∴ Resultant of force, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

∴ Direction of resultant,

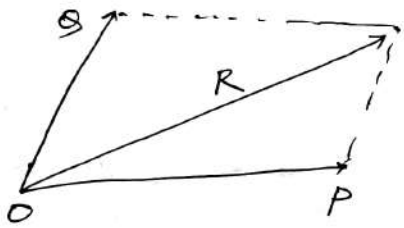
$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

(2) Graphical method

(a) Triangle law of force :

It states " If two force acting on a particle be represented in magnitude and direction by the two sides of a triangle, taken in order then resultant represented in magnitude & direction by the third side of triangle, taken in opposite order.".



Let two force P & Q acting at point 'O' & it's magnitude & direction represented two sides of triangle.

So, According to theorem of triangle of force, resultant will be represented by diagonal.

(b) polygon law of forces :

It states " If a number of forces acting on a particle, be represented in magnitude and direction by the sides of a polygon taken in order then the resultant of all these forces may be represented in magnitude & direction by the closing side of the polygon, taken in opposite order."

Moment of force :

* It is the turning effect produced by a force on which it acts.

* The moment of a force is equal to the product of the force and perpendicular distance of the point about which moment is required.

Mathematically,

$$m = P \times L$$

where, P = Force acting on the body.

l = perpendicular distance between the point.

* Unit of moment is kN-m, N-mm etc.

Types of moment :

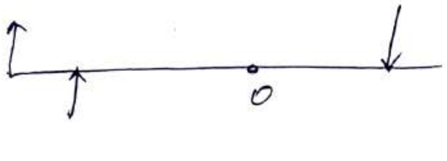
Moment

- Clockwise moment
- Anticlockwise moment

Clockwise moment

* It is the moment of force whose effect is to turn or rotate the body about in the same direction of hands of clock.

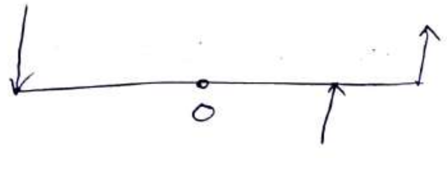
*



Anticlockwise moment

* It is the moment of force whose effect is to turn or rotate the body about in the same direction of hands of clock.

*



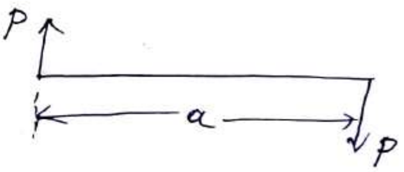
Varignon's principle of moment (Law of moment).

It states " If number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the force about any point is equal to the moment of their resultant force about the same point."

Couple :

* A pair of two equal and unlike parallel forces is known as couple.

* moment of couple = P x a



where, P = magnitude of the force.
a = Arm of couple.

Couple

- Clockwise couple
- Anticlock-wise couple

Clockwise couple

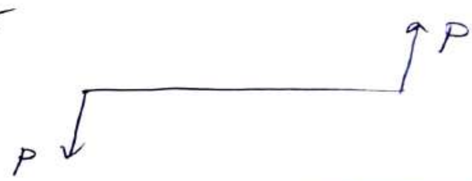
* A couple whose effect is to rotate the body in clockwise direction is called clockwise couple.

*

Anticlock-wise couple

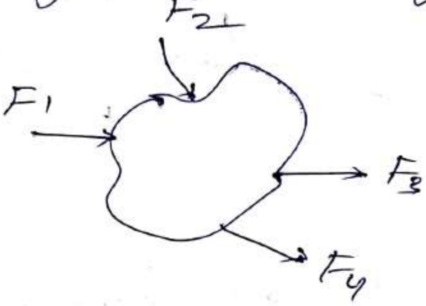
* A couple whose effect is to rotate the body in anticlockwise direction is called anti-clockwise moment.

*



Equilibrium of forces

A body can be said to be equilibrium when all the forces acting on a body balance each other or in other words there is no net force acting on the body.

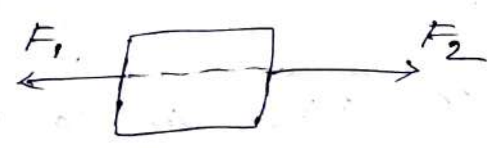


$\sum F = 0$

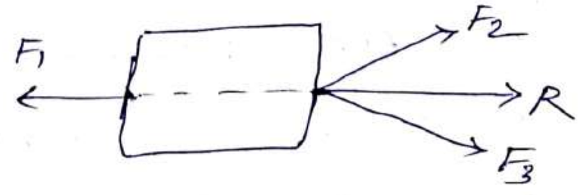
$\Rightarrow F_1 + F_2 - F_3 - F_4 = 0$

Principles of equilibrium :

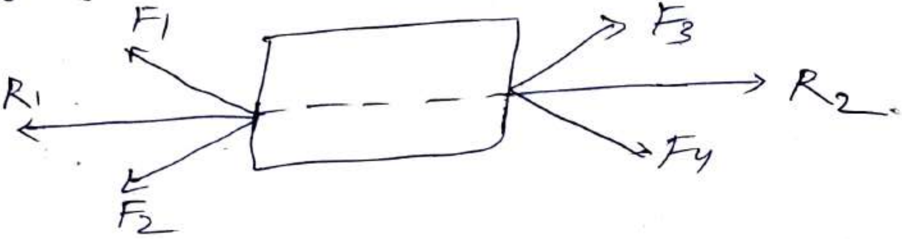
① Two force principle - A body in equilibrium is acted upon by two forces, then the forces must be equal, opposite & collinear.



② Three force principle : A body in equilibrium is acted upon by three forces, then the resultant of any two force must be equal, opposite and collinear with the third force.



(3) Four force principle : A body is in equilibrium is acted upon by four forces, then the resultant of any two force must be equal, opposite & collinear with the resultant of other two force.



Analytical conditions of equilibrium :

We know, Resultant of a system of co-planar concurrent force.

$$R = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

If the force are in equilibrium, $R = 0$

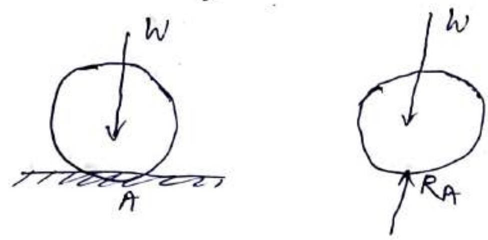
i.e $\Sigma x = 0$

$\Sigma Y = 0$

Free body diagram :

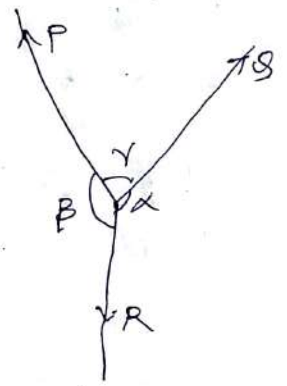
Free body diagram of a body is the diagram drawn by showing all the external force & reaction on the body and by removing the contact surface.

Diagram of free body :



Lami's theorm :

It states " If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two.



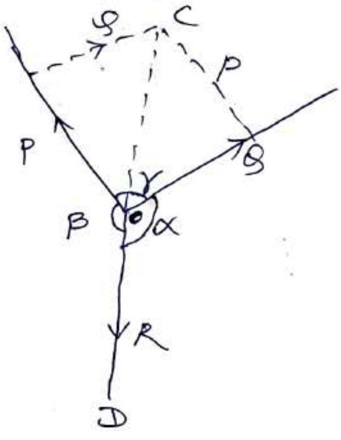
mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Proof

consider three co-planar forces P, Q & R acting at a point O.

Let opposite angles to three forces be α, β & γ.



complete the parallelogram OACB with OA & OB as adjacent.

From geometry of the figure.

$$PC = B \text{ and } AC = Q$$

$$\angle AOC = (180^\circ - \beta), \; \angle ACO = \angle BOC = (180^\circ - \alpha)$$

$$\therefore \angle CAO = 180^\circ - (\angle AOC + \angle ACO)$$

$$= 180^\circ - [(180^\circ - \beta) + (180^\circ - \alpha)]$$

$$= 180 - 180^\circ + \beta - 180^\circ + \alpha$$

$$= \alpha + \beta - 180^\circ$$

AS, $\alpha + \beta + \gamma = 360$

Now substituting 180° from both sides of above equation,

$$(\alpha + \beta - 180^\circ) + \gamma = 360^\circ - 180$$

$$\Rightarrow \angle CAO = 180^\circ - \gamma$$

we know, In triangle AOC.

$$\frac{OA}{\sin(\angle ACO)} = \frac{AC}{\sin(\angle AOC)} = \frac{OC}{\sin(\angle CAO)}$$

$$\Rightarrow \frac{OA}{\sin(180^\circ - \alpha)} = \frac{AC}{\sin(180^\circ - \beta)} = \frac{OC}{\sin(180^\circ - \gamma)}$$

$$\Rightarrow \boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

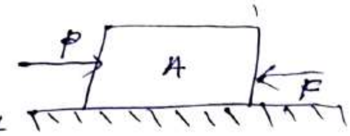
Friction

Friction: Friction is the resistance to motion of one object moving relative to another.

Limiting frictional force :

Limiting friction is the maximum amount of force, that comes in to play when a body just begins to move over the other surface.

Let, pushing force 'p' is acting to object 'A' but the object in not moving. It means the force (P) is less that frictional force (F). i.e $P < F$.



But, if more force applied then object start to move. It means force (P) is more than frictional force (F) i.e $P > F$.

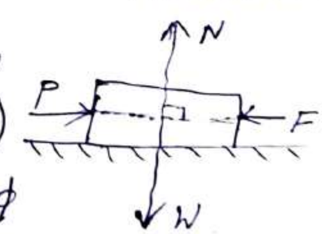
Coefficient of friction :

The ratio between the limiting friction & normal reaction of the two contact surface is called coefficient of friction.

So, friction force is directly proportional to normal reaction.

Hence, $F \propto N$. (N = Normal reaction)

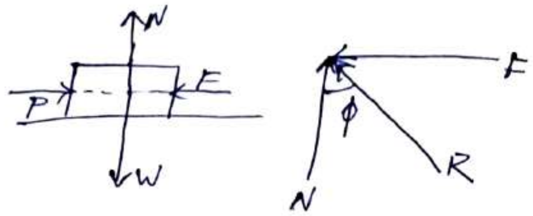
$\Rightarrow F = \mu N \Rightarrow \mu = \frac{F}{N} = \tan\phi$



Angle of friction :

The angle of friction is the angle between the normal reaction & the resultant of frictional force & reaction.

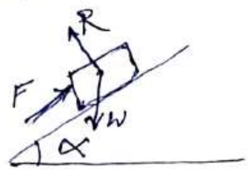
$$\mu = \frac{F}{N} = \tan\phi .$$



Angle of repose :

It is an angle of the inclined plane at which the body is tends to slide downwards.

It is specified by α.



Laws of friction :

There are two laws of friction.

(1) Laws of static friction.

(2) laws of dynamic friction.

(1) Laws of Static friction :

(1) The force of friction always acts opposite of the applied force.

(2) The magnitude of frictional force is exactly equal to the applied force.

(3) The Ratio of limiting friction to normal reaction bears constant value between two surface

(4) The Force of friction is independent of area contact between two surface.

(5) The force of friction depends upon the roughness of the surface.

② Laws of dynamic friction :

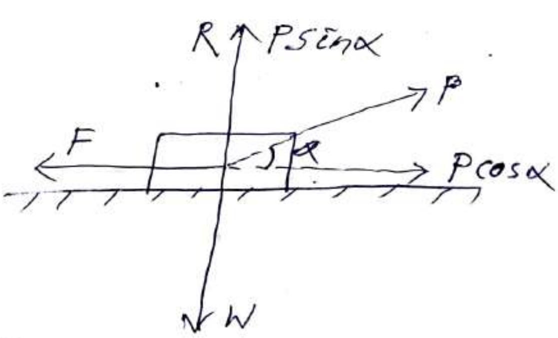
① The force of friction always act in direction opposite to the body is moving.

② The ratio of kinetic friction to normal reaction between two surface bears a constant value

* this ratio slightly less than in case of limiting friction.

③ For moderate speed the force of friction is remains constants but decrease slightly with increase of speed.

Angle made by the force with the horizontal



(*) Resolving force vertically.

$$R = W - P\sin\alpha$$

(*) Resolving force horizontally

$$F = P\cos\alpha$$

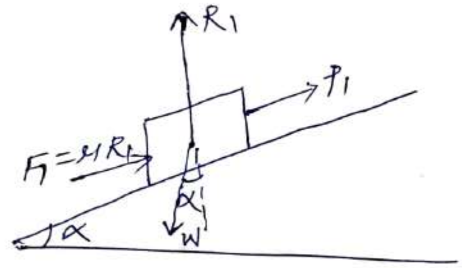
(*) We know, force of friction.

$$F = \mu R$$

① Equilibrium of a body on a Rough Inclined plane subjected to a force acting along the inclied plane :

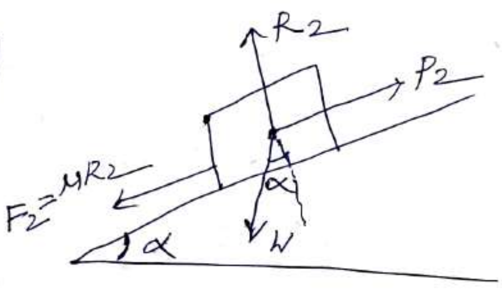
(i) Minimum force (P_1) required to keep the body in equilibrium, when the body sliding downwards.

$$P_1 = W \times \frac{\sin(\alpha - \phi)}{\cos \phi}$$



(ii) Maximum force (P_2) required to keep the body in equilibrium, when the body sliding downwards

$$P_2 = W \times \frac{\sin(\alpha + \phi)}{\cos \phi}$$



Where, W = weight of the body

α = Angle, which the inclied plane makes with the horizontal.

R = Normal reaction

μ = Coefficient of friction between the body.

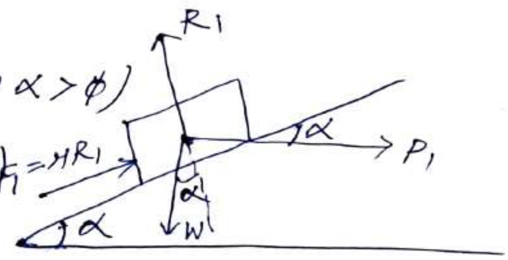
ϕ = Angle of friction.

(2) Equilibrium of a body on a rough inclied plane subjected to a force acting horizontally.

(i) minimum force (P_1) required to keep the body in equilibrium, when the body sliding downward

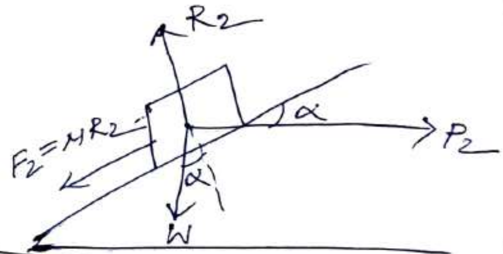
$P_1 = W \tan(\alpha - \phi)$ ....(when $\alpha > \phi$)

$= W \tan(\phi - \alpha)$ ...(when $\phi > \alpha$)



(ii) maximum force (P_2) required to keep the body in equilibrium when the body sliding upwards.

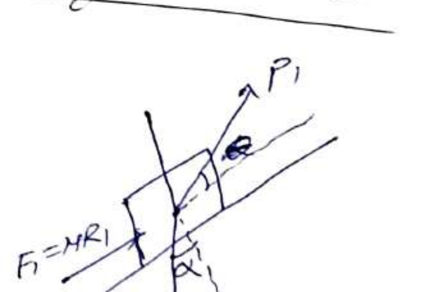
$P_2 = W \tan(\alpha + \phi)$



(3) Equilibrium of a body on a rough inclined plane subjected to a force acting at some angle with the inclined plane

(i) minimum force (P_1) required to keep the body in equilibrium when the body sliding downwards.

$$P_1 = W \times \frac{\sin(\alpha - \phi)}{\cos(\theta + \phi)}$$



similarly, For max. force (P_2).

$$P_2 = W \times \frac{\sin(\alpha + \phi)}{\cos(\theta - \phi)}$$

Application of friction :

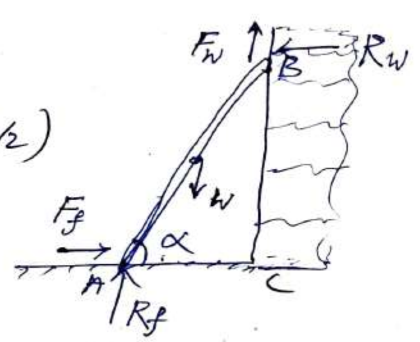
Following are the important where friction applies the most.

1. Ladder friction.
2. Wedge friction.
3. Screw friction.

① Ladder friction :

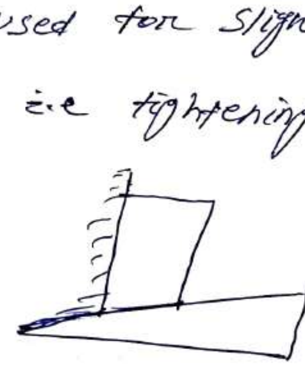
The ladder is a device for climbing on the roof or wall.

$F_f \times BC = (R_f \times AC) - (W \times AC/2)$



② Wedge friction :

A wedge is a triangular or trapezoidal in cross-section; It is generally used for slight adjustment in the position of a body i.e tightening fit or keys.



③ (Extra) Screw friction :

Pitch : The distance from one point of thread to the corresponding point on the next thread.

Lead

It is the distance through which a screw thread advance axially in one turn.

Lead = Pitch × No. of thread.

Slope of Thread :

It is the inclination of the thread with horizontal.

$$\tan\alpha = \frac{\text{Lead of Screw}}{\text{Circumference of screw}}$$

α = Angle of inclination $= \dfrac{p}{\pi d}$ (For single thread)

p = pitch of screw

$= \dfrac{np}{\pi d}$ (multi thread)

d = mean dia. of screw.

n = No. of threads

Relation between weight & Effort by a screw jack

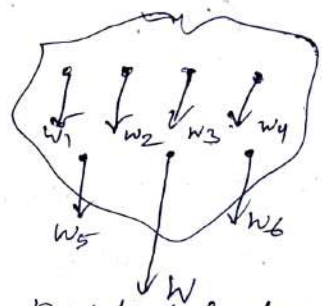
$$\tan\alpha = \frac{p}{\pi d}, \quad P = W\tan(\alpha+\phi)$$

$$\mu = \tan\phi$$

Centroid and moment of Inertia

Centre of gravity (C.G):

The centre of gravity of a body is the point at which the entire weight of the body act irrespective of the position and orientation of the body.



(*) A body is made up number of particles having weights w_1, w_2, w_3 --- which are attracted towards the centre of the body.

The resultant of all these forces at a point also called Centre of gravity (C.G).

Centroid:

Centroid is the centre point or geometric centre of a plane like triangle, circle, square etc.

Method for centre of grawity:

1. By geometrical consideration.
2. By moments.
3. By graphical method.

NOTE

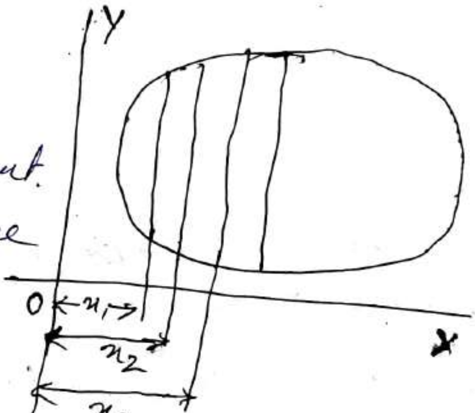
(*) Symmetrical section → Two halves that exactly match each other. when symmetrical about x-x axis only calculate $\bar{x}$.

(*) If symmetrical about y-y axis then calculate $\bar{y}$.

(*) But for asymmetrical section calculate both $\bar{X}$ & $\bar{Y}$.

Centre of gravity by moments :

* Consider a body of mass (m) whose C.G required to be calculate.
* Divide the body into small masses i.e m_1, m_2, m_3 ---- and the co-ordinates from point 'O' are (x_1, y_1) (x_2, y_2) (x_3, y_3) etc .



From the principle of moment.
Let, $\bar{x}$ & $\bar{y}$ be the C.G of the body.

As we know, $m\bar{x} = m_1x_1 + m_2x_2 + m_3x_3 + \cdots$

$$\Rightarrow \bar{x} = \frac{\Sigma mx}{m}$$

Similarly, $$\bar{y} = \frac{\Sigma my}{m}$$

Where, $m = m_1 + m_2 + m_3 + \cdots$

Centre of gravity of plane figure :

The centre of gravity of plane geometrical figure is known as centroid & coincides with C.G.

Let, $\bar{x}$ & $\bar{y}$ be the co-ordinates of C.G with respect to some axis of reference.

$$\bar{x} = \frac{a_1x_1 + a_2y_2 + a_3y_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}$$

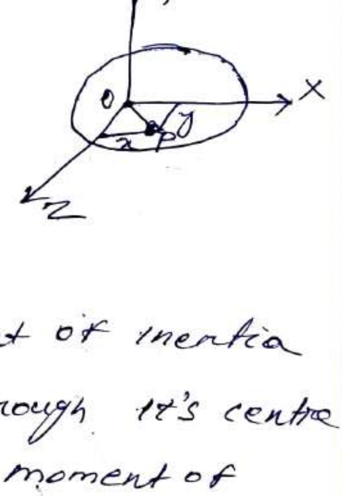
Moment of Inertia :

Moment of inertia is defind as the quantity expressed by the body resisting angular acceleration which is the sum of the product of the mass of every particle with it's square distance from it's axis of rotation.

Theorem of perpendicular axis :

It states " If I_{xx} and I_{yy} be the moment of Inertia of a plane section about two perendicular axis meeting at O, the moment of Inertia I_{zz} about axis Z-Z perpendicular to the plane & passing through the intersection of X-X and Y-Y axis. is given by :

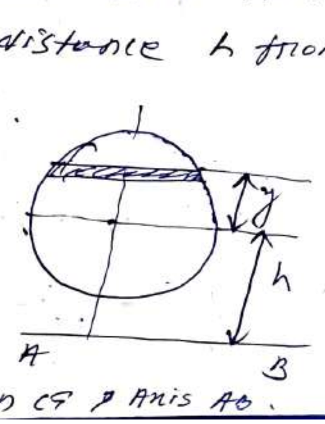
$$I_{zz} = I_{xx} + I_{yy}$$



Theorem of parallel axis :

It states, If the moment of inertia of a plane area about an axis through it's centre of gravity is denoted by I_G, then moment of inertia of the area about any other axis AB. parallel to the first and at a distance h from the centre of gravity is given by.

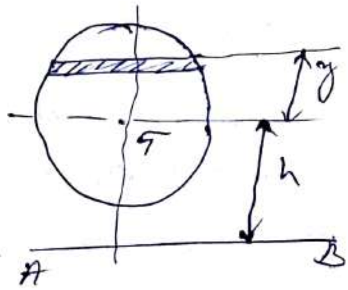
$$I_{AB} = I_G + ah^2$$



where, I_{AB} = M.I of area from axis AB.
I_G = M.I of the area it's C.G.
a = Area of section, h = Distance between CG & Axis AB.

Proof of Parallel axis theorem :

Consider a strip of a circle, whose M.I is required to be found.



Let, δA = Area of strip.

y = Distance from strip to C.G

h = Distance between C.G and axis AB.

We know, moment of inertia of the strip about centre of gravity of section.

$$\delta a \cdot y^2 \qquad \left(I = \text{area} \times (\text{distance}^2) \right)$$

$\therefore$ moment of inertia of the section about axis AB.

$$I_{AB} = \sum \delta a (h+y)^2$$

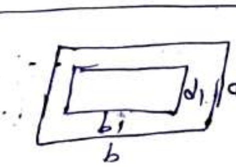
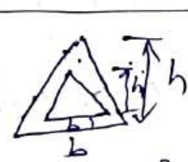
$$= \sum \delta a (h^2 + y^2 + 2hy)$$

$$= \left(\sum h^2 \cdot \delta a\right) + \left(\sum y^2 \cdot \delta a\right) + \left(\sum 2hy \cdot \delta a\right)$$

$$= ah^2 + I_G + 0$$

$$\Rightarrow \boxed{I_{AB} = I_G + ah^2}$$

Formula :

 $I_x = \frac{bh^3}{12}, I_y = \frac{hb^3}{12}$	 $I_x = \frac{bh^3}{36}, I_y = \frac{hb^3}{36}$	 $I_x = I_y = \frac{\pi}{64} d^4$
 $I_x = \frac{bh^3}{12} - \frac{b_1 h^3}{12}, I_y =$	 $I_x = \frac{bh^3}{36} - \frac{b_1 h^3}{36}$,	 $I_x = I_y = \frac{\pi}{64}(d^4 - d_1^4)$

Simple machine

Simple machine: A simple m/c is a device by which load can be lifted by applying less effort as compared to the load.

Ex- Gear, pulley, wedge, screw, wheel & axle, Lever etc.

Compound machine:

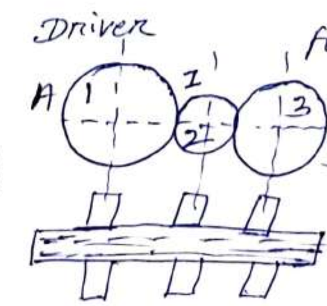
Compound machine is a device which consist of number of simple machines.

Ex- Rack & Pinion, worm wheel Differential screw jack.

Simple gear train:

* In simple gear train each shaft support one gear.

* A simple gear drive all the gear lie in same plane. In below diagram show a simple gear drive in which gear A and gear B lie in the same plane.

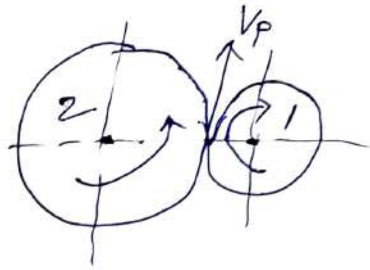


(fig. No-1)

* The gear 'A' is driver gear & gear 'B' is follower gear and 'I' is Idle gear.

* The function of Idle gear is to fill the gap & and change the direction of motion.

Simple gear drive :



Gears Use to transmit power from shaft to another Shaft.

Vp = Tangential velocity at point of contact of two gear.

$V_P = \omega_1 r_1 = \omega_2 r_2$

$\Rightarrow \pi d_1 N_1 = \pi d_2 N_2 \Rightarrow \frac{N_1}{N_2} = \frac{d_2}{d_1}$

Velocity ratio of a simple gear train :

From fig. 1, N_1 = speed of driver.
T_1 = No. of teeth on driver
d_1 = Diameter of driver.
N_2, T_2, d_2 = corresponding values for intermediate gear.
N_3, T_3, d_3 = corresponding values for follower

Pitch of driver, $p = \frac{\pi d_1}{T_1}$ --- (1)

Similarly, $p = \frac{\pi d_2}{T_2}$ --- (2)

$p = \frac{\pi d_3}{T_3}$ --- (3)

As, pitch of matting gear are same.
Equating equation (2) & (3), Equating equ (3) & (4)

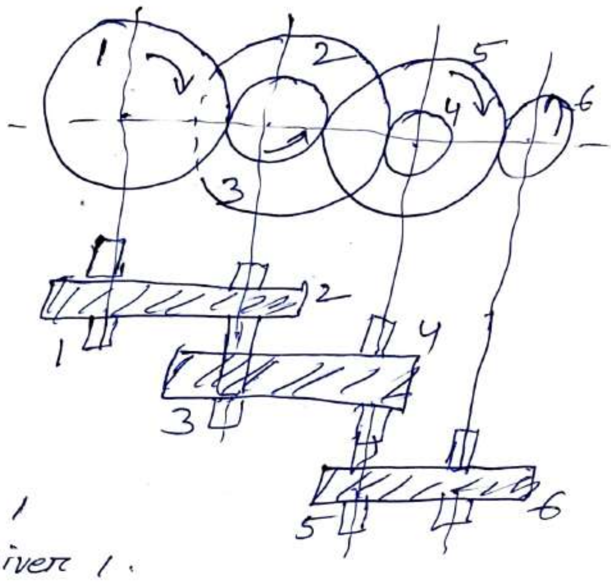
$\frac{d_1}{d_2} = \frac{T_1}{T_2}$, $\frac{d_2}{d_3} = \frac{T_2}{T_3}$

As, we know. $\frac{N_1}{N_2} = \frac{d_2}{d_1}$

So, $\frac{N_1}{N_2} = \frac{T_2}{T_1}$ and $\frac{N_2}{N_3} = \frac{T_3}{T_2}$, By multiplying $\boxed{\frac{N_1}{N_3} = \frac{T_3}{T_1}}$

Compound gear train :

When a series of gear are connected in such a way that two or more gears are mounted on same shaft is know as compound gear train.



N_1 = speed of driver 1

T_1 = No. of teeth on driver 1.

Similarly. N_2, N_3, N_4, N_5, N_6 = speed of respective wheel.

T_2, T_3, T_4, T_5, T_6 = No. of teeth on respective wheel.

As gear (1) and (2) mesh with each other.

Therefore, $\frac{N_1}{N_2} = \frac{T_2}{T_1}$ --- (1)

Similarly. $\frac{N_3}{N_4} = \frac{T_4}{T_3}$ --- (2)

$N_5/N_6 = \frac{T_6}{T_5}$ --- (3)

multiplying equation (1). (2) & (3).

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$$\Rightarrow \frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5} = \frac{\text{Product of teeth on follower}}{\text{Product of teeth on driver}}$$

(As, $N_2 = N_3$, $N_4 = N_5$)

Effort The force which is applied to lift load.

Load The weight to be lifted with the help of m/c.

Velocity ratio :

$$V.R = \frac{\text{Distace travelled by effort}}{\text{Distance travelled by load}}$$

Mechanical advantage :

$$M.A = \frac{\text{Load}}{\text{Effort}} = \frac{W}{P}$$

Efficiency

The ratio of output to input.

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{Amount of workdone by load}}{\text{Amout of w/d. by effort}}$$

$$= \frac{W \times x}{P \times y}$$

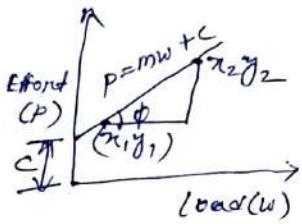
$$= \frac{M.A}{V.R}$$

So, $\boxed{\eta = \frac{M.A}{V.R}}$

Law of machine :

The equation which gives the relation between load lifted and effort applied in the form of slope and intercept of a straight line called law of m/c.

$$P = mW + c$$



Where, P = Effort applied
W = Load lifted
m = slope of the line.
c = y-intercept of the straight line.

$$m = \tan\phi = \frac{y_2 - y_1}{x_2 - x_1}$$

Maximum mechanical advantage

We know, $M\cdot A = \frac{W}{P} = \frac{W}{mW + c} = \frac{1}{m + \frac{c}{W}}$

$$\Rightarrow \boxed{M\cdot A = \frac{1}{m}}$$

(Neglecting $\frac{c}{W}$)

Maximum efficiency :

$$\therefore \eta_{max} = \frac{M\cdot A_{max}}{V\cdot R} = \frac{1}{m} \times \frac{1}{V\cdot R} = \frac{1}{m \times VR}$$

Ideal machine :

If the efficiency of a machine is 100 % ; the m/c is called Ideal machine.

Reversible of a machine :

(*) When a machine is also capable of doing some work in the reverse direction after the effort is removed. Such m/c called reversible machine

(*) For reversible of a m/c the efficiency should be more than 50%. i.e $\boxed{\eta > 50\%}$

Non-reversible or self-locking machine :

(*) When a m/c is not capable of doing any work in the reverse direction, after effort is removed, such condition called non-reversible or self locking machine.

(*) For self locking the efficiency is not more than 50%. i.e $\eta < 50\%$

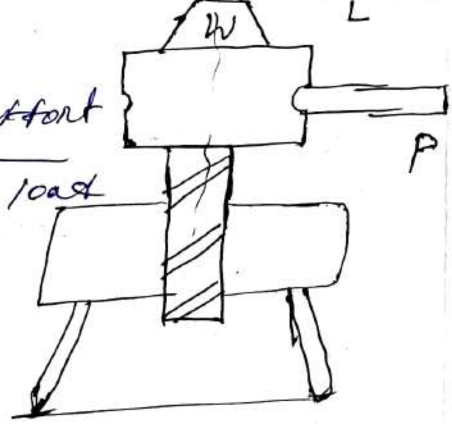
Screw jack :

* A screw jack is commonly used for lifting and supporting heavy load. A very small effort can be applied at the end of the lever.

* When the effort is applied to lever arm to complete one revolution then load is lifted through one pitch of screw.

Therefore the distance moved by the load is equal to the pitch of the screw & distance moved by the effort is equal to $2\pi l$.

Let, L, P, W, P.



(1) $V \cdot R = \dfrac{\text{Distance travelled by effort}}{\text{Distance travelled by load}}$

$= \dfrac{2\pi L}{P}$

(2) $M \cdot A = \dfrac{W}{P}$

(3) $\eta = \dfrac{M \cdot A}{V \cdot R}$

Dynamics

Kinematics: It is the branch of dynamic which deals with motion of bodies without considering the forces causing motion.

Kinetics: It deals with mothion of the bodies & the force causing motion. It predict type of motion by a given force system.

Principle of Dynamics:

Newton's Law of motion

① First law of motion:

The law states that "Everybody continues in it's state of rest or of uniform motion in a straight line, unless acted by some external force to change that state."

② Second law of motion:

It states " The rate of change of momentum is directly proportional to the impressed force & takes place in the same direction in which impressed force acts".

$$\text{Rate of change of momentum} = \frac{mv - mu}{t}$$

$$= \frac{m(v-u)}{t} = ma.$$

According to law.

$$F \propto ma$$

$$\Rightarrow F = k \cdot ma \quad , k = \text{constant proportionality.}$$

③ Third law of motio:

It state" To every action there is alway equal & opposite reaction."

Equation of motion :

Following are the equation of motion.

$$V = u + at$$
$$S = ut + \frac{1}{2}at^2$$
$$V^2 - u^2 = 2as$$

Where, u = Initial velocity of body.
v = Final Velocity of body.
s = Distance travelled by the body in motion
a = Acceleration of the body.
t = Time taken by the body.

D' Alembert's Principle :

It states " If a rigid body is acted by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction & the line of action may found out by method of graphic statics".

So, Force acting on a body is

$$P = ma \quad \text{---- (i)}$$

Where, m = mass of the body.
a = Acceleration of the body.

The equation (i) may also written as.

$$P - ma = 0$$

The equation (i) is equation of dynamics whereas equation (ii) is equation of Statics.

Recoil of gun :

According to newton's 3 rd law of motion, when a bullet is fired, the opposite reaction of the bullet is known as recoil of gun.

Let, m_1 = mass of gun

V_1 = velocity of gun.

m_2 = mass of the bullet.

V_2 = velocity of the bullet after firing.

momentum of the bullet after firing $= m_2V_2$

momentum of gun $= m_1V_1$

So, Equating equation ① & ② we get.

$$m_1V_1 = m_2V_2$$

This relation is known as "Law of conservation of momentum".

Work :

* When force acts on a body and the body undergoes some displacement, the work is said to be done.

* The amount of work done is equal to the product of force & displacement in the direction of force.

$$\text{Work done } (W) = P \times S$$

(Unit N-m, KN-m Joule.)

where, P = Force acting on the body.

S = Distance through which body moves.

When; force & displacement are not collinear.

$$W = P\cos\theta \times S$$

Power :

"Power is defined as the rate of doing work".

$$\text{Power} = \frac{\text{Workdone}}{\text{Time}} \quad \left(\text{Unit } \frac{N\text{-}m}{sec} \text{ or } J/sec\right)$$

Energy :

"Energy may be defined as capacity of doing work"..

As energy is measured by work, therefore unit of energy is same as work.

There are two types of mechanical energy

① Potential energy -

The energy possessed by virtue of it's position.

If a body of mass (m) is raised to a height (h) above ground level. The work done in raising the body.

= weight of body × Distance it moved.

$= mgh$

The above work is stored in the body termed as potential energy. $PE = mgh$

② Kinetic energy :

The energy possessed by a body by virtue of it's motion

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

momentum :

The product of mass and velocity of the body. It represent the energy of motion stored in a moving body.

momentum of body = mV (unit kg.m/sec)

Impulse :

Impulse is defind as the product of force & time during which the force acts on the body.

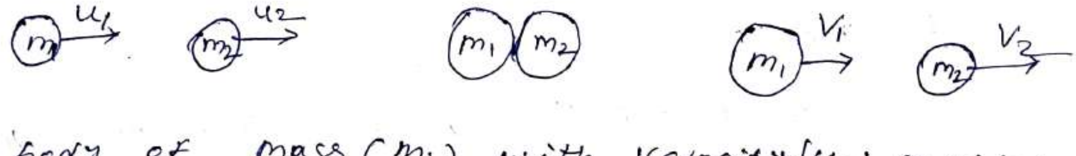
$$\text{Impulse}(I) = F \times t$$
$$= ma \times t$$
$$= mv - mu$$

i.e change in momentum is equal to impulse.

Law of conservation of linear momentum :

It states "the total momentum of two bodies remains constant after their collision or any other mutual action."

momentum along a straight line i called linear momentum.



Let a body of mass (m_1) with velocity (u_1) collides with another body (m_2) mass moving with velocity (u_2). If V_1 & V_2 be velocity of bodies after collision.

Total momentum before collision = $m_1u_1 + m_2u_2$

Total momentum after collision = $m_1V_1 + m_2V_2$

Now, According to the law of conservation of linear momentum.

⇒ momentum before collision = momentum after collision

⇒ $m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$

Newton's law of collision of elastic bodies :

It states " When two moving bodies collides with each other, their velocity separation bears a constant ratio to their velocity approach.

mathematically, $$(V_2 - V_1) = e(u_1 - u_2)$$

where, e = coefficient of restitution.
u_1, V_1 = intial & final velocity of 1st body.
u_2, V_2 = intial & final velocity of 2nd body.

Types of collision :

When two bodies collide with each other they said to have impact. Threre are two types of impact.

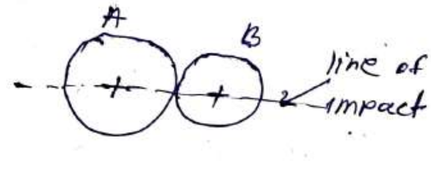
① Direct Impact

② Indirect impact (Oblique)

Direct impact

If two bodies, before impacting moving along the line of impact, is called direct impact.

consider two bodies A & B having direct impact.



m_1, m_2 = mass of bodies A & B respectively.
u_1, u_2 = initial velocity of bodies A & B.
V_1, V_2 = Final velocity of bodies A & B.

AS, we already discussed.

$$\boxed{m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2}$$