A LECTURE NOTE
ON
TH.1 THEORY OF MACHINE
SEMESTER -4

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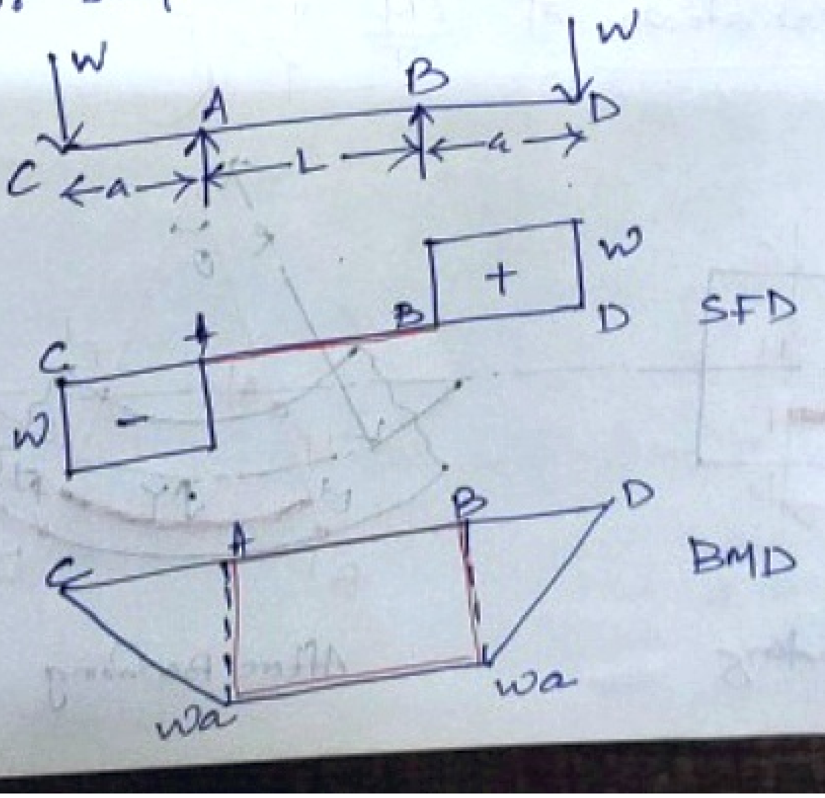
Bending Stresses in Beams :-

When some external load acts on a beam, the shear force and Bending Moment are set up at all section of beam. Due to SF & BM the beam undergoes certain deflection. The material of the beam will offer resistance or stresses against these deformation. These stresses with certain assumptions can be calculated. The stresses introduced by bending moment are known as bending stresses.

Pure Bending / Simple Bending :-

If a length of a beam subjected to a constant bending moment and no shear force (zero shear force) then the stresses will be set up in that length of the beam due to BM only and that length of the beam is said to be in pure bending or simple bending



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Theory of Simple Bending:-

1) The material of the beam is homogeneous and isotropic.

2) The value of Young's modulus of elasticity is the same in tension and compression.

3) The transverse section which were plane before bending, remain plane after bending also.

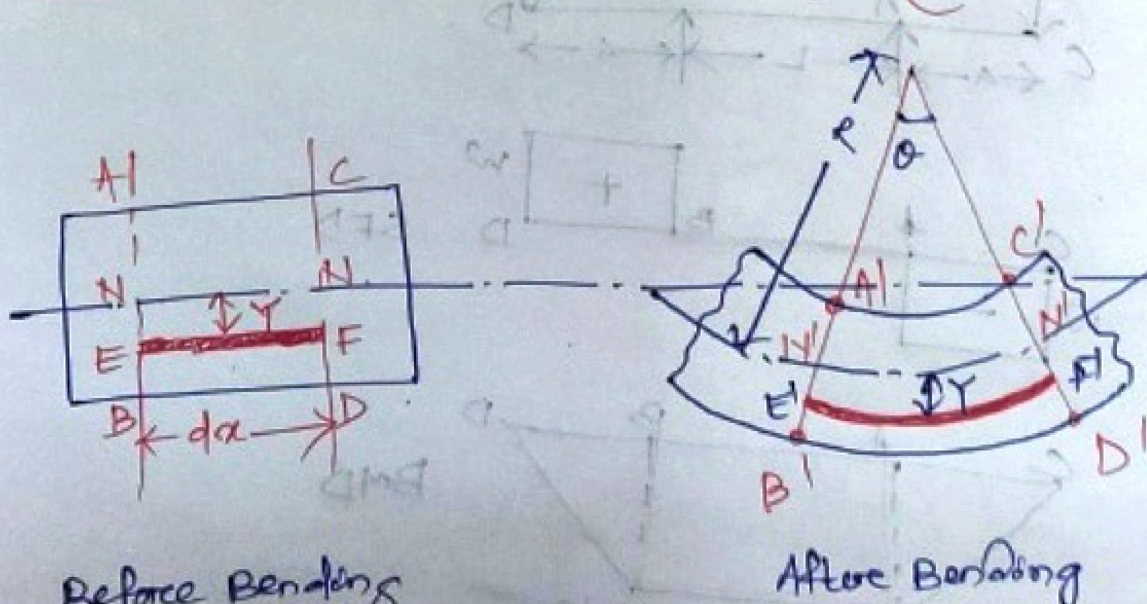
4) The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.

5) The radius of curvature is large compared with the dimensions of the cross-section.

6) Each layer of the beam is free to expand or contract independently of the layers, above or below it.

Theory of simple Bending/

Derivation of $\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}$



Before Bending   After Bending

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Let R = radius of neutral layer $N'N'$
θ = Angle subtended at O by $A'B'$ & $C'D'$

Original length of $EF = dx$.

$$AC = BD = NN = EF = dx \longrightarrow \langle 1 \rangle$$

After Bending.

$$N'N' = NN = dx \longrightarrow \langle 2 \rangle$$

$$N'N' = R \times \theta \longrightarrow \langle 3 \rangle$$

$$E'F' = (R + y)\theta \longrightarrow \langle 4 \rangle$$

∴ Increase in the length of the layer EF

$$= E'F' - EF$$

$$= (R + y)\theta - R\theta$$

$$= \cancel{R\theta} + y\theta - \cancel{R\theta}$$

$$= y\theta \longrightarrow \langle 5 \rangle$$

∴ Strain in the layer EF $= \dfrac{\text{Change in length}}{\text{Original length}}$

$$= \frac{y\theta}{R\theta}$$

$$e = \frac{y}{R}$$

As we know young's Modulus $E = \dfrac{\sigma}{e}$

$$e = \frac{\sigma}{E} \longrightarrow \langle 6 \rangle$$

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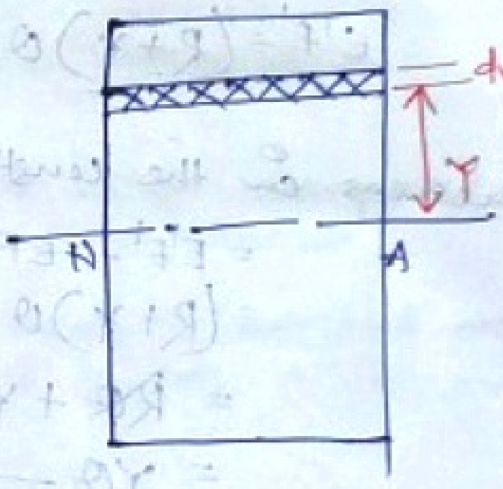
$$\boxed{\frac{y}{R} = \frac{\sigma}{E}} \longrightarrow \langle 7 \rangle$$

Neutral Axis and Moment of Resistance :-

The neutral axis of any transverse section of a beam is defined as the line of intersection of the neutral layer with the transverse section. It is written as N.A.



$$\sigma = \frac{y}{R} \cdot E$$

Force on the layer = stress × area

$$= \sigma \times dA$$

$$= \frac{E}{R} \cdot y \cdot dA \longrightarrow \langle 8 \rangle$$

Total force on the beam $= \int \frac{E}{R} \times y \times dA$

$$= \frac{E}{R} \int y\, dA .$$

$$\frac{E}{R} \int y\, dA = 0 .$$

$$\int y\, dA = 0$$

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Force on layer = $\frac{E}{R} \times y \times dA$

Moment of this force about NA = Force × y.

$$M = \frac{E}{R} \times y \times dA \times y$$

$$M = \frac{E}{R} \times y^2 dA$$

$$M = \int \frac{E}{R} y^2 dA$$

$$M = \frac{E}{R} \int y^2 dA \quad \text{—— ⟨9⟩}$$

$$M = \frac{E}{R} \times I$$

$$\frac{M}{I} = \frac{E}{R} \quad \text{—— ⟨10⟩}$$

$$\boxed{\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}}$$ Bending Equation

M is expressed in Nmm; I in mm^4
σ is expressed in N/mm^2; Y in mm
E is expressed in N/mm^2; R in mm