A LECTURE NOTE
ON
CIRCUIT AND NETWORK THEORY (TH-2)

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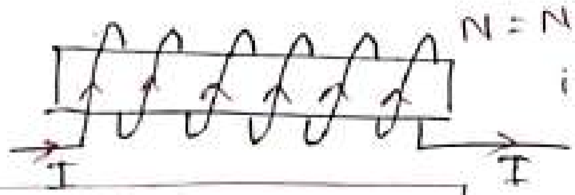
Magnetic circuits

The path of the magnetic flux is called the magnetic circuit

Definations concerning magnetic circuit

1. Magnetomotive Force (MMF)

It drives or tends to drive flux through a magnetic circuit & corresponds to electromotive force emf in an electric circuit.



N = No. of turns
i = current in coil

MMF = NI
Unit of MMF = Ampere - turn (AT)

2. Magnetic Flux (Ø)

The total No. of lines of force existing in a particular magnetic field is called magnetic flux.

Symbol → Ø
Unit → weber (Wb)

3. magnetic Flux density (B)

It is defined as the flux per unit Area in a plane at right angles to each other.

$$B = \frac{\phi}{A} = \frac{Wb}{m^2} \text{ or Tesla}$$

Unit - Wb/m^2 or Tesla
Symbol - Ø

Ø = total flux (in Wbs)
A = Area perpendicular to the lines in (m^2)

4) Magnetic Field strength (H) / Magnetic Field intensity

→ Magnetic Field intensity (H) at any point in the magnetic field is defined as the force experienced by the unit North pole at that point.

→ magnetic Field strength of a circuit is given by mmf per unit length.

$$\boxed{H = \frac{NI}{l} \; \frac{AT}{m}} \quad \boxed{\text{unit } (AT/m)}$$

(5) Reluctance (S)

→ It is the property of a ~~material~~ medium which opposes the passage of magnetic flux.

reluctance (S) = $\frac{mmf}{\phi}$ or $S = \frac{NI}{\phi}$ (AT/wb)

$\boxed{\text{unit } At/wb}$

(6) Permeance

→ it is a measure of readiness with which flux is developed.

→ It is reciprocal of reluctance $\frac{1}{S} = \frac{\phi}{NI}$

(7) permeability (μ)

→ Permeability of a material is the measure of the ease, with which magnetic lines of force pass through a given material.

→

→ The magnetic flux density (B) and it's intensity (H) in a medium can be related by the following eqn.

$\boxed{B = \mu H}$ or $\boxed{\mu = \frac{B}{H}}$

where $\boxed{\mu = \mu_0 \mu_r}$ is permeability of the medium H/m.

μ_0 = Absolute permeability of free space $= 4\pi \times 10^{-7}$ H/m

μ_r = ~~rela~~ relative permeability of the medium

$$\cancel{\mu = \frac{B}{H} = \frac{\phi/A}{NI/l} = \frac{\phi}{A} \times \frac{l}{NI}}$$

unit	μ =	Henry/m
	μ_r =	unit less
	μ_0 =	H/m

Relative permeability (μ_r) is a non-dimensional numeric which indicates the degree to which the medium is a better conductor of magnetic flux as compared to free space.

$\mu_r = 1$ For air and non-magnetic material

$\mu_r = 1000 - 10{,}000$ For ferro-magnetic material

⊗ S(Reluctance) $= \frac{mmf}{\phi} = \frac{NI}{\phi}$ At/wb $= \frac{NI}{l} \times \frac{l}{\phi}$

$$\boxed{\because H = \frac{NI}{l},\ B = \frac{\phi}{A} \\ B = \mu H}$$

$$= \frac{NI}{l} \times \frac{l}{BA}$$

$$= \frac{H}{B \cancel{l}}$$

(((--

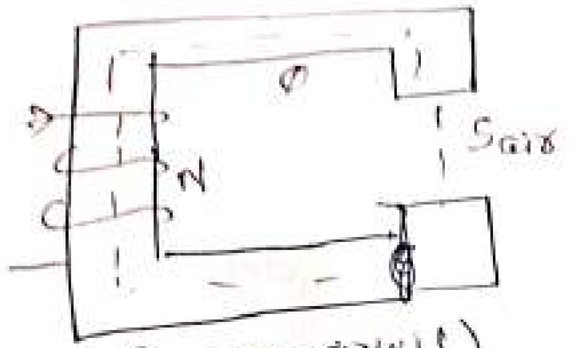
$$\boxed{S = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A} \quad AT/wb}$$

Reluctance of magnetic circuit depends upon

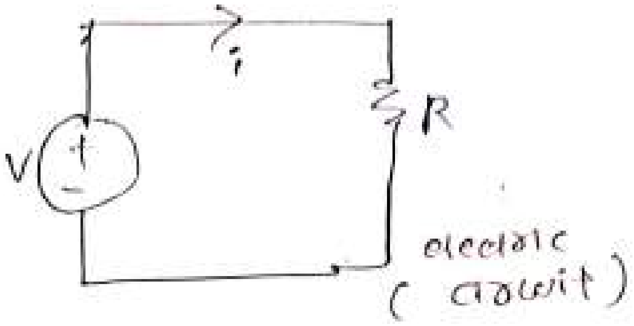
- Nature of the material i.e. $\mu_0 \mu_r$
- Length of the magnetic path i.e. 'l'
- cross-sectional area of the material i.e. 'a'.

Analogy b/w Electric & magnetic circuit

Magnetic circuit	Electric circuit
① MMF = magneto-motive Force unit = Ampere-turn (AT)	① emf = electromotive Force unit = Volt (V)
② Flux (ϕ) unit (wb)	② current (i) unit = Amp
③ Flux = $\frac{\text{mmf}}{\text{reluctance}} = \frac{Ni}{S}$	③ current (i) = $\frac{\text{emf}}{\text{resistance}} = \frac{V}{R}$
④ $R = \rho \frac{l}{a}$ (Resistance)	④ $S = \frac{l}{\mu a}$ (reluctance)
⑤ Flux density (B) = $\frac{\phi}{A}$	⑤ current density $J = \frac{A}{m^2}$
⑥ magnetic Field Intensity (H) = $\frac{\text{mmF}}{l} = \frac{Ni}{l}$ (AT/m)	⑥ Electric Field Intensity $E = \frac{V}{d}$ (Volt/m)
⑦ permeance = $\frac{1}{\text{reluctance}}$	⑦ conductance = $\frac{1}{\text{resistance}}$
⑧ Total emf $= IR_1 + IR_2 + IR_3 \ldots$	⑧ Total e.m.f $= \phi S_1 + \phi S_2 + \phi S_3 + \ldots$



(magnetic circuit)

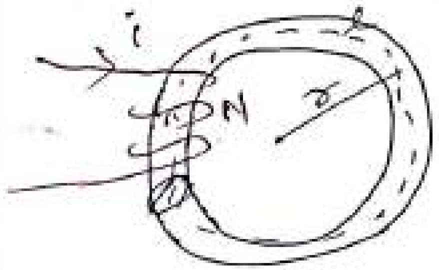


Magnetic circuits

(1) Series magnetic circuit

A series magnetic circuit is analogous to a series electric circuit.

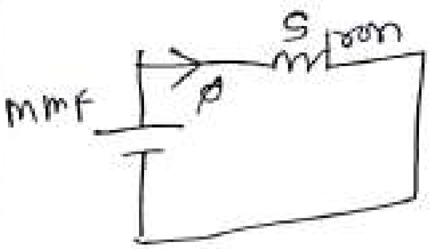
Consider a ~~ring Toroid~~ ring having a magnetic path of 'l' meters, area of cross-section (A) m^2 with a mean radius of 'r' meter having a coil of 'N' turns carrying 'I' ampers wound uniformly,



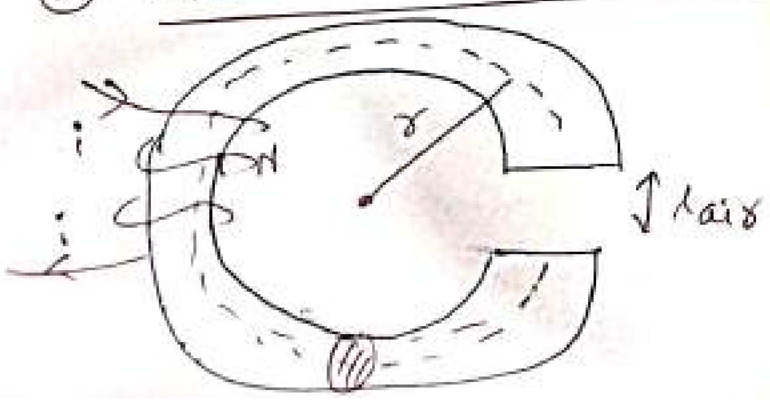
$l = 2\pi r$ → mean radius

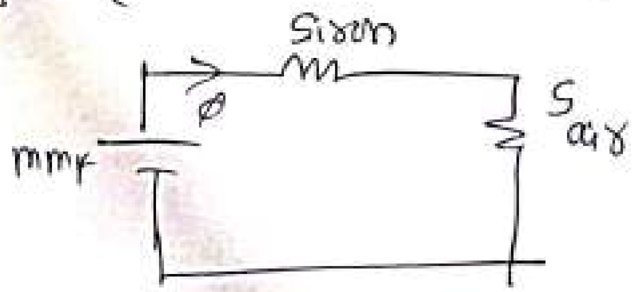
$H = \frac{NI}{l}$ At/m , $B = \mu_0 \mu_r H$

$\phi = \frac{NI}{(l/\mu_0 \mu_r A)}$ ab $= \frac{NI}{S}$



(2) Composite magnetic circuit (Toroid with Air gap)





$mmf = \phi S_{total}$
$= \phi (S_{iron} + S_{air})$

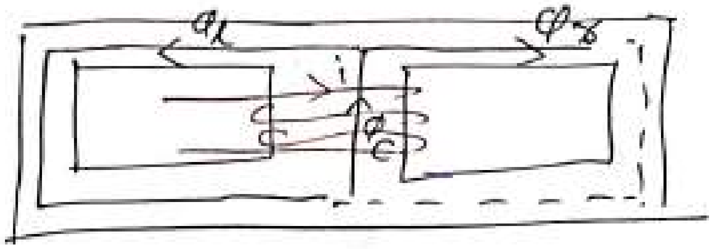
$Ni = mmf_{iron} + mmf_{air}$

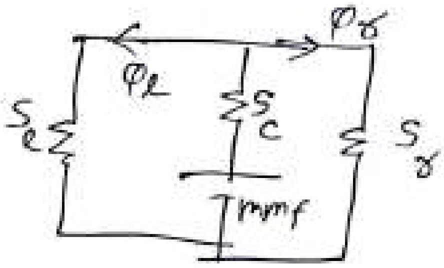
$\downarrow S_{iron} = \frac{l_{iron}}{\mu_0 \mu_r a}$ low, $\uparrow$ high

$\uparrow S_{air}^{high} = \frac{l_{air}}{\mu_0 (1) a}$ $\uparrow$ low

$l = l_{iron} + l_{air}$, $l = 2\pi r$ $B = \mu_0 \mu_r H$

③ <u>Parallel magnetic circuit</u>





$\phi_c = \phi_r + \phi_l$

$Ni - \phi_c S_c = [\phi_l S_l] = [\phi_r S_r]$

in parallel circuit mmf is same .

B-H curve

i) The curve which shows the relationship b/w the magnetising force(H) and magnetic flux density of a magnetic substance is known as the B-H curve.

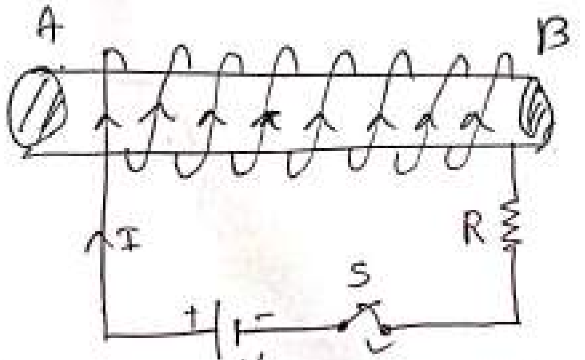
ii) B = Flux density i.e. $B = \frac{\phi}{A}$ wb/m²

H = the magnetising force $= \frac{NI}{l}$

Let the coil has N turns and l, the length which is constant in this particular case,

so $H \propto I$

iii)



→ Let the specimen AB be an unmagnetised soft iron piece, which is wound.

→ The magnetising force(H) can be increased or decreased by increasing ~~the~~ or decreasing the current through the solenoid.

→ When I = 0, H = 0 and Flux density (B) is initially zero.

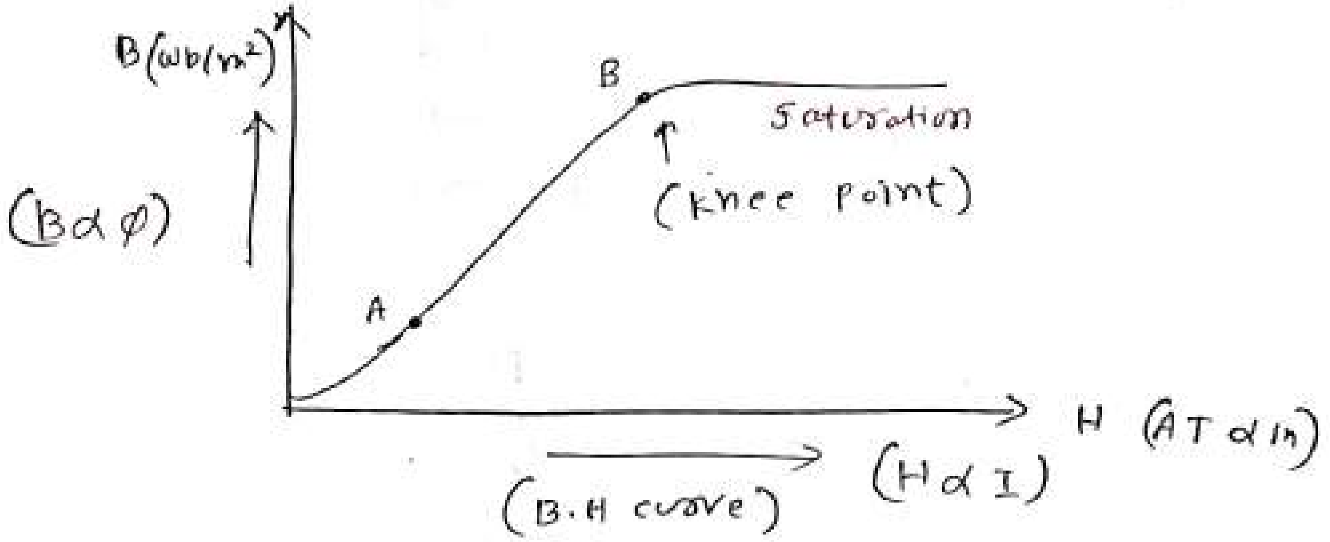
→ As H i.e. I is increased the Flux density(B) also increases.

→ The rate is higher at the starting time &

and rate gradually decreases with the increase of current.

→ There is one stage where flux density doesn't increase even when the current is increased, that is known as saturation point.

iv) If these B vs H is plotted in a curve

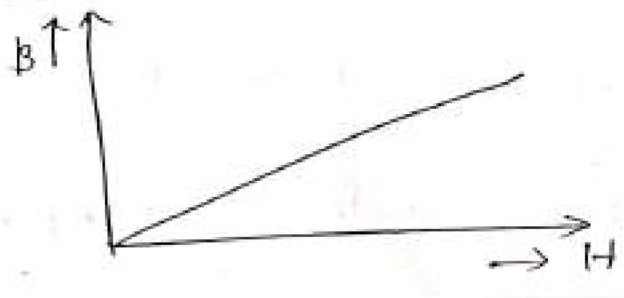


(B.H curve)

v) Uses of B-H curve

1. It is used to find out the saturation point of the magnetic material, so it is useful for designing purpose.
2. It is used to find out the permeability i.e. B/H of the material.

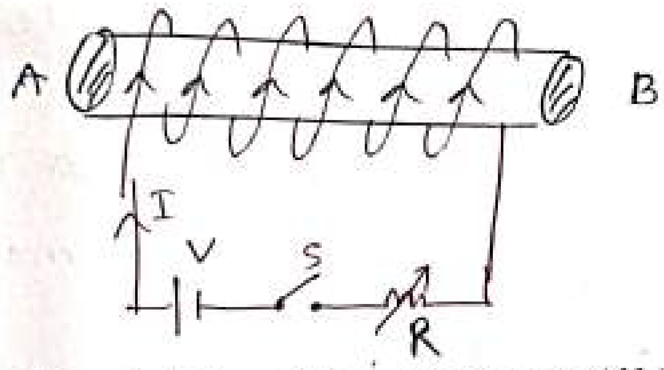
B-H curve of a Non-magnetic material



Hysteresis loop

Magnetic Hysteresis

i) The phenomenon of lagging of magnetisation or induction flux density (B) behind the magnetising force (H).



ii) Let us take an unmagnetised bar of iron (AB) and magnetise it by placing it within the field of a solenoid. The field ($H = NI/l$) produced by the solenoid is called the magnetising force.

iii) The value of 'H' can be increased or decreased by increasing or decreasing current through the coil. Let H be increased in steps from zero up to a certain maximum value and the corresponding values of flux density (B) be noted.

iv) If we plot the relation b/w H & B, a curve like OA is obtained. The material becomes magnetically saturated for H = OM and has at that time a maximum flux density of B_{max} established through it.

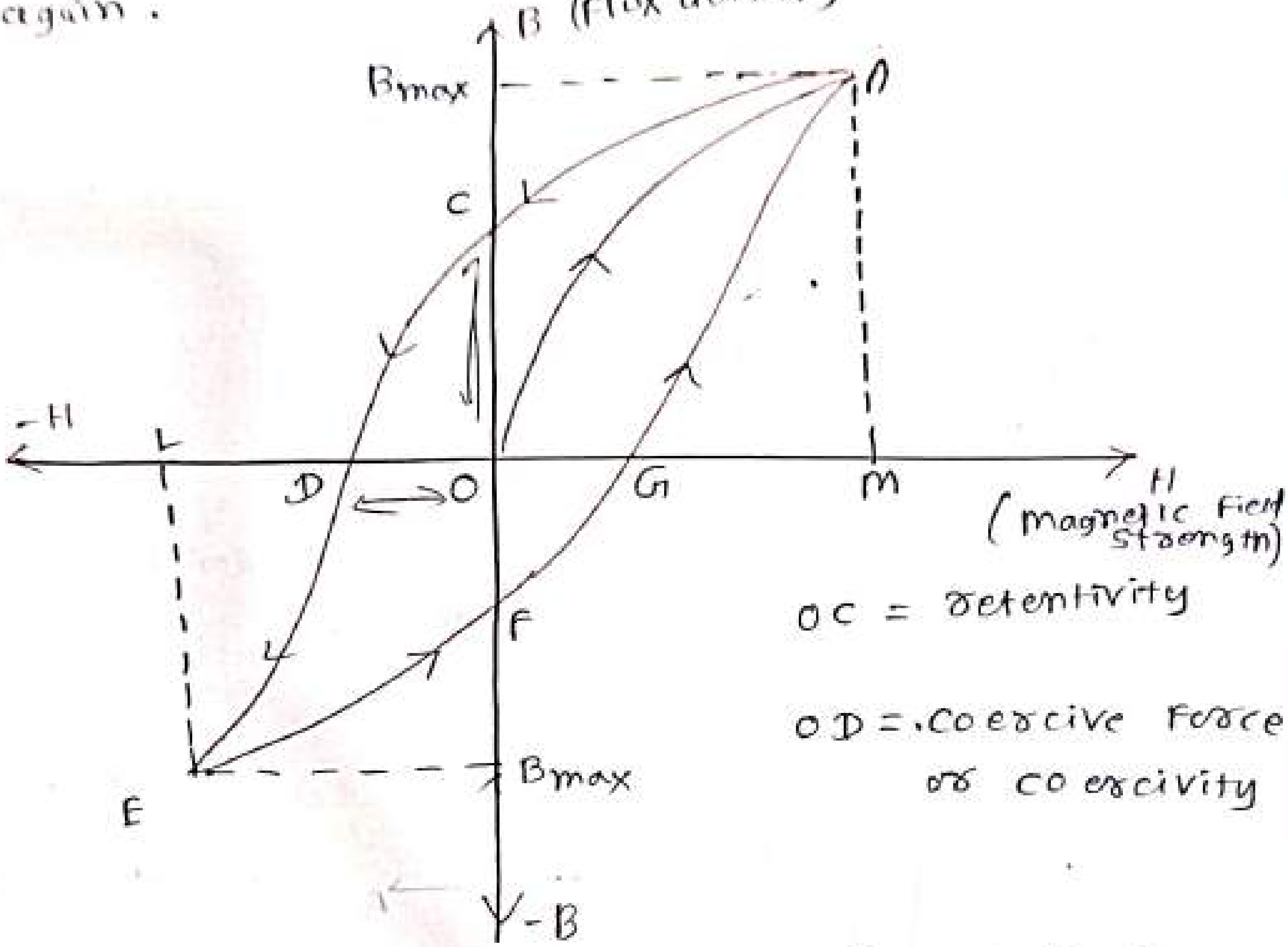
v) If H is now decreased gradually (by decreasing solenoid current), Flux density B will not decrease along AO, as might be expected, but will decrease less rapidly along AC.

vi) When $H = 0$, $B \neq 0$, $B = OC$. It means that on removing the magnetising force H, the iron Bar is not completely demagnetised. This value of B (= OC) measures the retentivity or remanence of the material & is called the remanent or residual Flux density (B_r).

vii) To demagnetise the iron bar, we have to apply the demagnetising force in the reverse direction. When H is reversed (by reversing current through the solenoid) then 'B' is reduced to zero at point 'D' where $H = OD$. This value of H required to wipe off residual magnetisation or magnetism is known as coercive force (H_c) & is a measure of the coercivity of the material i.e. it's tenacity with which it holds onto its magnetism.

viii) If, after the magnetisation has been reduced to zero, the value of H is further increased in the negative i.e. reversed direction, the iron bar again reaches a state of magnetic saturation, represented by point 'K'.

by taking H back from its value corresponding to negative saturation (= OL) to its value for positive saturation (= OM), a similar curve FFGA is obtained. if we again start from 'G', the same curve GACDEFG is obtained once again.



OC = retentivity

OD = coercive force or coercivity

It is seen that B always lag behind H. The B & H never attain zero simultaneously.

→ This lagging of 'B' behind 'H' is given the name 'hysteresis' which means to lag behind.

→ The closed loop ACDEFGA which is obtained when iron bar is taken through one complete cycle of magnetisation is known as 'hysteresis loop'.

Area of a hysteresis loop

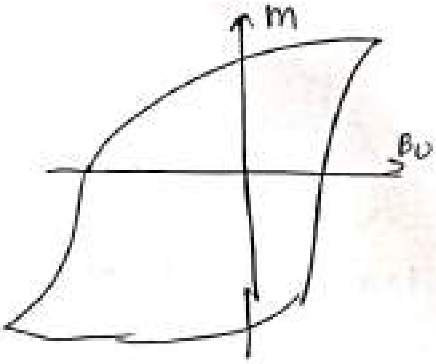
→ The area of the hysteresis loop represents the net energy spent in taking the iron bar through one cycle of magnetisation.

→ Hysteresis loop measures the energy dissipated due to hysteresis which appears in form of heat & so raises the temperature of that portion of the magnetic circuit which is subject to magnetic reversal.

W_h = (Area of B/H loop) joule m^3/cycle

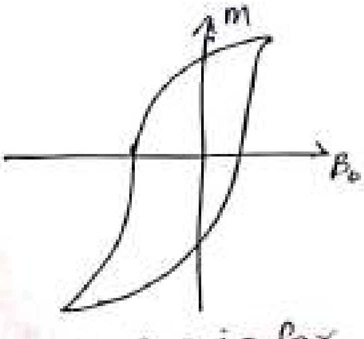
↳ hysteresis loss.

(A) The shape of the hysteresis loop depends on the nature of the magnetic material.



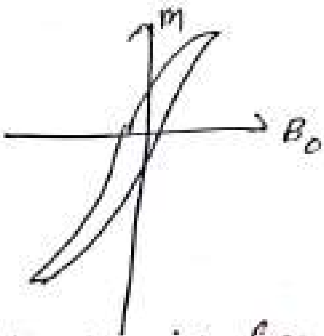
loop-1 is For hard steel.

Uses : making permanent magnet



loop-2 is for wrough iron & cast steel.

Uses : used for making electromagnet



loop-3 is for alloyed sheet steel

Used: armature & transformer core

② Coupled circuits

Self induced e.m.f

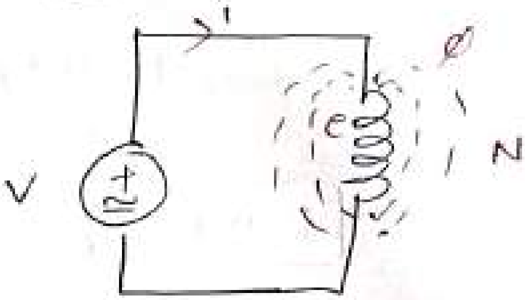
→ This is the e.m.f induced in a coil due to the change of its own flux linked with it.

→ If current through coil is changed, flux linked with its own turns will also change, which will produce in it what is called self-induced e.m.f.

Self-inductance

The property of the coil due to which it opposes any increase or decrease of flux through it is known as self-inductance.

Co-efficient of self-induction (L)



$$\phi = \phi \frac{i}{i}$$

$$\phi = \left[\frac{\phi}{i}\right] i$$

$$\frac{d\phi}{dt} = \left(\frac{\phi}{i}\right)\frac{di}{dt} \quad ..(i)$$

$$e = -N\frac{d\phi}{dt} \quad ..(ii)$$

$$e = \left(-\frac{N\phi}{i}\right)\frac{di}{dt} \Rightarrow \boxed{L = \frac{N\phi}{i} = \frac{\psi}{i}}$$

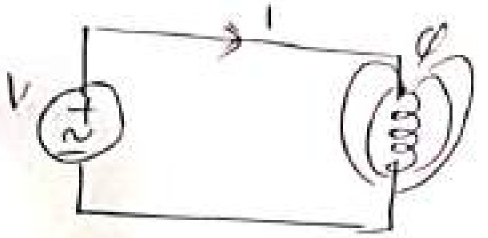
$$\boxed{e = -L\frac{di}{dt}}$$

self induced emf (in volts)

L = self inductance (H)

→ Consider a coil consisting of N turns and carrying a current 'i' in clockwise direction. If the current is steady, then the magnetic flux through the closed path will remain constant.

→ However, suppose the current 'i' changes with time then according to Faraday's law, an induced emf will arise to oppose the change.



The property of a closed path in which its own magnetic field opposes any change in current is called self inductance and the emf generated is called the self induced emf.

$$L = \frac{N\phi}{i} \text{ Henry} = \frac{N\left(\frac{mmf}{S}\right)}{I} = \frac{N\left(\frac{NI}{l/\mu A}\right)}{I}$$

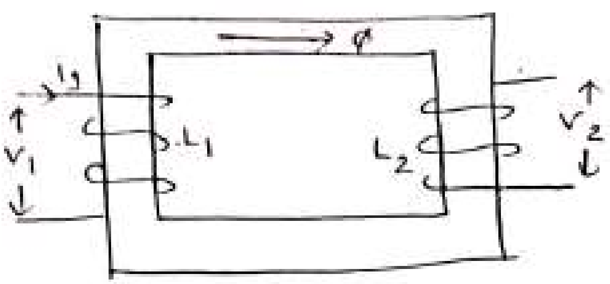
$$\boxed{\therefore L = \frac{N^2 \mu A}{l}} \text{ Henry}$$

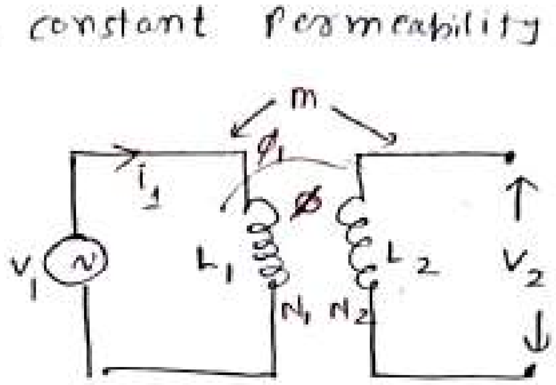
$$\boxed{L \propto N^2}$$

Concept of mutual inductance

→ Let us consider two coils L_1 & L_2 which are sufficiently close to each other, so that the flux produced by i_1 in coil L_1 also link coil L_2.

→ We assume that the coils do not move with respect to one another, and the medium in which the flux is produced has constant permeability.





$\phi_1 = \phi_{11} + \phi_{12}$

ϕ_1 → total flux; ϕ_{11} → leakage flux; ϕ_{12} → common flux or mutual flux

Two coils or circuits are said to be inductively coupled, because of this property they are called coupled elements.

→ The time varying flux surrounding the second coil L_2 induces an emf or voltage across the terminals of L_2; this voltage is proportional to the time rate of change of current flowing through the first coil.

$\cancel{V_2(t) = M_{21} \frac{di_1(t)}{dt}}$ $\quad \cancel{V_2(t) = M_{21} \frac{di_1(t)}{dt}}$ ~~volt~~

$\cancel{L_1 = \frac{N_1 \phi_1}{I_1}}$ ~~Henry~~ $\qquad M = \frac{N_2 \phi_{12}}{i_1}$

$\cancel{M_{21} = \frac{N_1 \phi_{21}}{}}$ $\qquad M = \frac{N_1 \phi_{21}}{i_2}$

→ similarly if current i_2 is made to pass through coil L_2 with coil L_1 open, a change of i_2 would cause a voltage in coil L_1.

if distance b/w coils & Permeability of medium b/w coils is constant, then

$$m_{12} = m_{21} = m$$

Voltage induced in coil '2' due to current in coil '1'

$$V_{21} = m\frac{di_1}{dt}$$

Voltage induced in coil '1' due to current in coil '2'

$$V_{12} = m\frac{di_2}{dt}$$

Total Voltage in voltage coil - 1

$$V_1 = \underbrace{L_1 \frac{di_1}{dt}}_{\text{induced voltage due to self inductance}} + \underbrace{m\frac{di_2}{dt}}_{\text{induced voltage due to mutual inductance}}$$

To get the relation b/w self & mutual inductance, co-efficent of coupling may be defined.

co-efficent of coupling

The degree to which the mutual inductance approaches its maximum value is given by co-effient of coupling.

$$\boxed{K = \frac{m}{\sqrt{L_1 L_2}} \qquad m = K\sqrt{L_1 L_2}}$$

m = mutual inductance between the coils

L_1 = self inductance of the 1st coil

L_2 = self inductance of the 2nd coil

but $0 \le K \le 1$, $\Rightarrow M \le K\sqrt{L_1 L_2}$

Q) Two inductively coupled coils have self inductances $L_1 = 50mH$ & $L_2 = 200 mH$. If the co-efficient of coupling is 0.5 i) Find the value of mutual inductance b/w the coils ?
ii) Find max^m possible mutual inductance ?

A - i) $M = K\sqrt{L_1 L_2}$
$= 0.5\sqrt{50\times10^{-3} \times 200\times10^{-3}} = 50\times10^{-3}\ H$

ii) max^m value of the inductance when $K = 1$
$M = \sqrt{L_1 L_2} = 100 mH$

Series connection of coupled inductors

dot convention

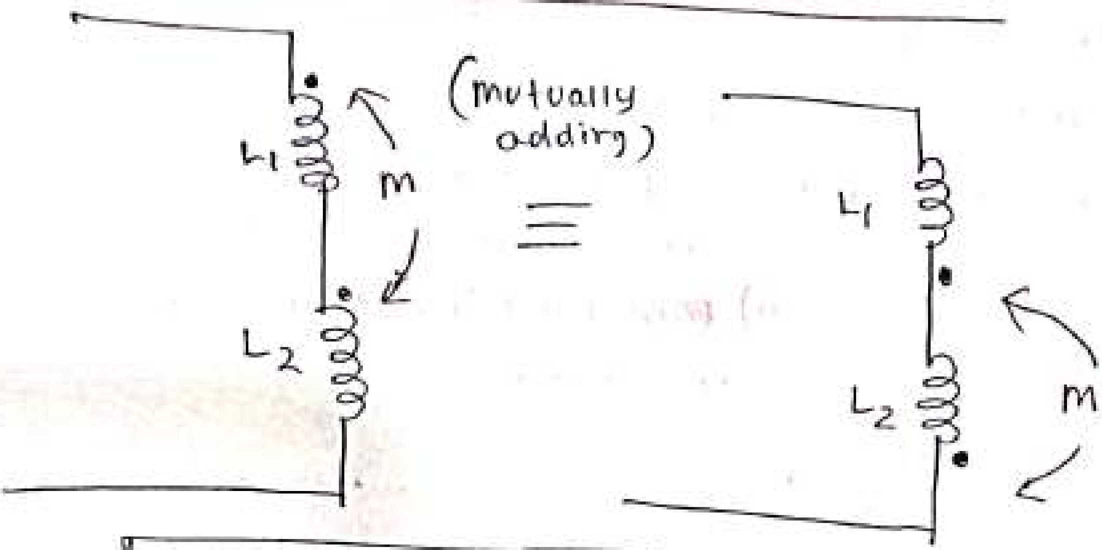
In circuit analysis, the dot convention is a convention used to denote the voltage polarity of the mutual inductance of two components

i) If a current enters the dotted terminal of one coil, then the polarity of induced emf induced in the second coil will be positive at the dotted terminal of the 2nd coil.

ii) If a current leaves the dotted terminal of one coil, then the polarity of the emf induced in the 2nd coil will be negative at the dotted terminal of the 2nd coil.

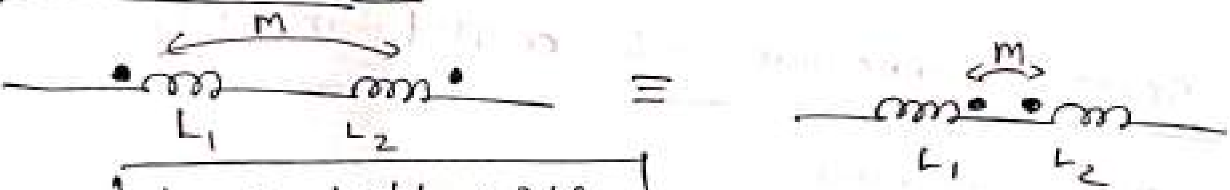
Series connection of coupled inductors

a)



$$\boxed{L_{eq} = L_1 + L_2 + 2M}$$

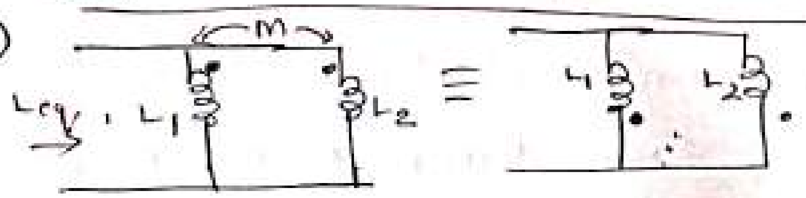
b) Mutually opposing



$$\boxed{L_{eq} = L_1 + L_2 - 2M}$$

Parallel connection of coupled coils

a)



mutually adding

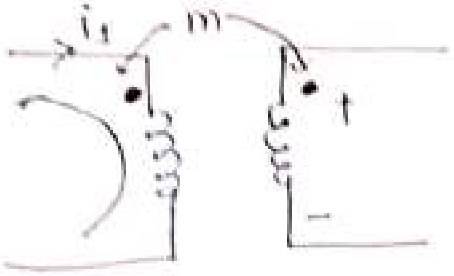
$$\boxed{L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}$$

b) Mutually opposing

$$\boxed{L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}}$$

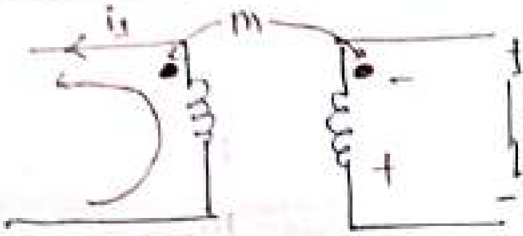
Q) determine the current polarity & magnitude of induced emf w.r.t. given reference for system of coils shown below?

i)



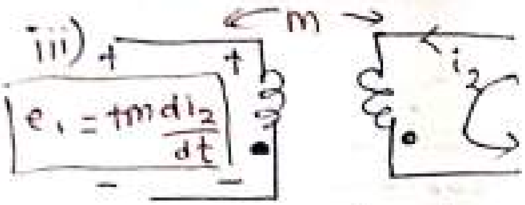
$$\boxed{e_2 = +m\frac{di_1}{dt}}$$

ii)



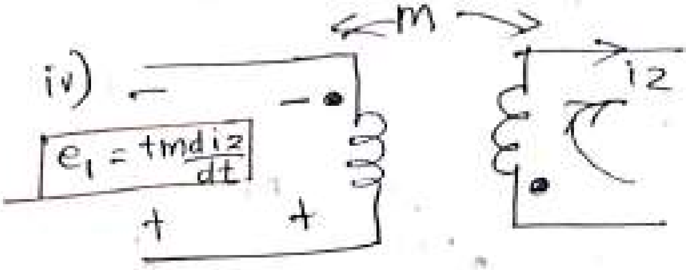
$$\boxed{e_2 = -m\frac{di_1}{dt}}$$

iii)



$$\boxed{e_1 = +m\frac{di_2}{dt}}$$

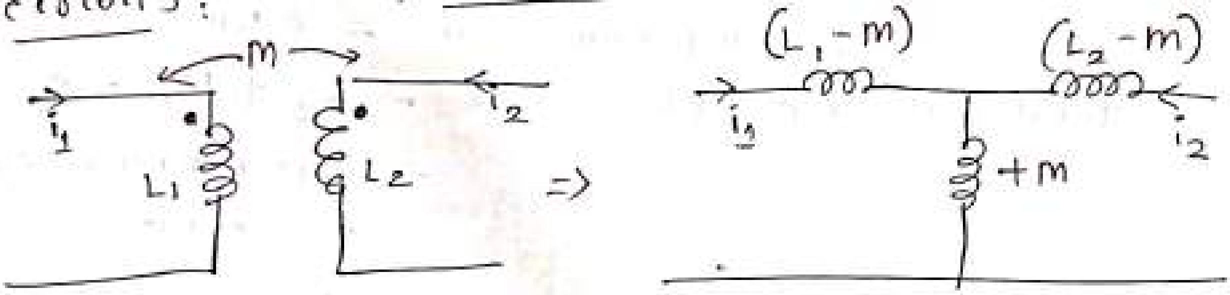
iv)



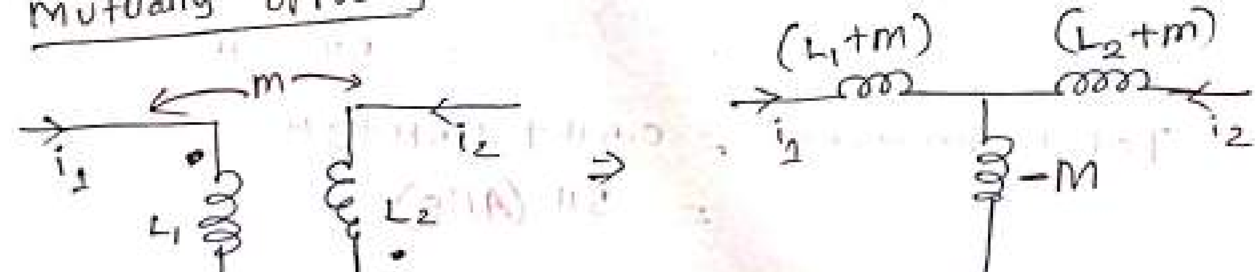
$$\boxed{e_1 = +m\frac{di_2}{dt}}$$

'T' equivalent representation of ideal T/F in circuits:

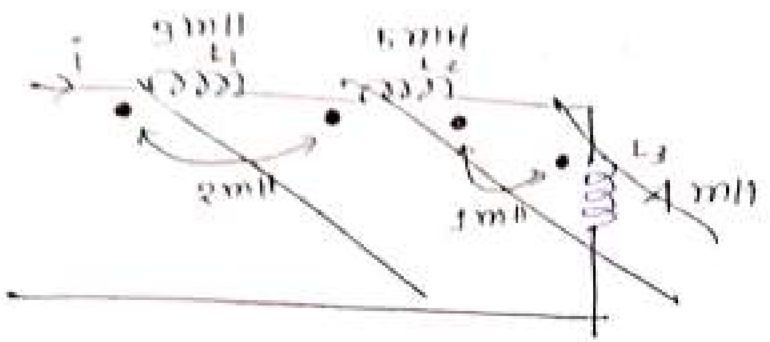
a) Mutually adding



b) Mutually opposing

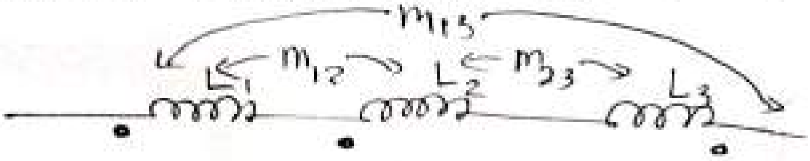


2) Q)



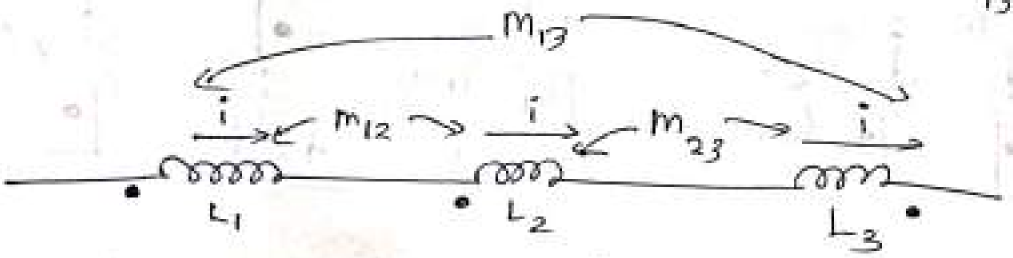
A

Q) Find the total inductance of three series connected coupled coil are shown



$L_1 = 1H$, $L_2 = 2H$, $L_3 = 5H$, $M_{12} = 0.5H$, $M_{23} = 1H$,
$M_{13} = 1H$

A :-



Total inductance of coil 1

$= L_1 + M_{12} - M_{13}$

$= 1H + 0.5H - 1H = 0.5H$

Total inductance of coil-2 $= L_2 + M_{12} - M_{23}$

$= 2H + 0.5H - 1H$

$= 1.5H$

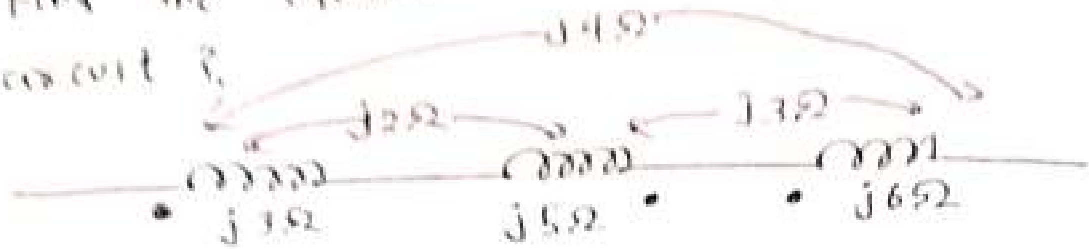
Total inducance of coil-3 $= L_3 - M_{23} - M_{13}$

$= 5H - 1H - 1H = 3H$

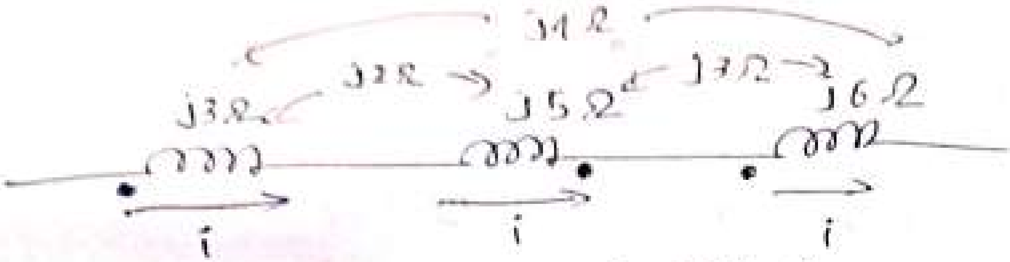
Total inductance $= 0.5H + 1.5H + 3H$

$= 5H$ (ANs)

Find the equivalent impedance of the shown circuit &



Ans:



<u>Total reactance of coil 1</u>

$$X_1 = j3 - j2 + j4 = j5\,\Omega$$

<u>Total reactance of coil 2</u>

$$X_2 = j5 - j3 - j2 = 0\,\Omega$$

<u>Total reactance of coil 3</u>

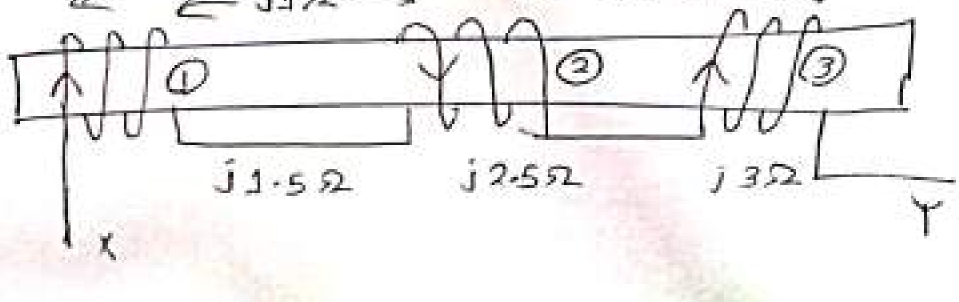
$$X_3 = j6 - j3\,\Omega + j4\,\Omega = j7\,\Omega$$

Total reactance of the circuit $= X_1 + X_2 + X_3$

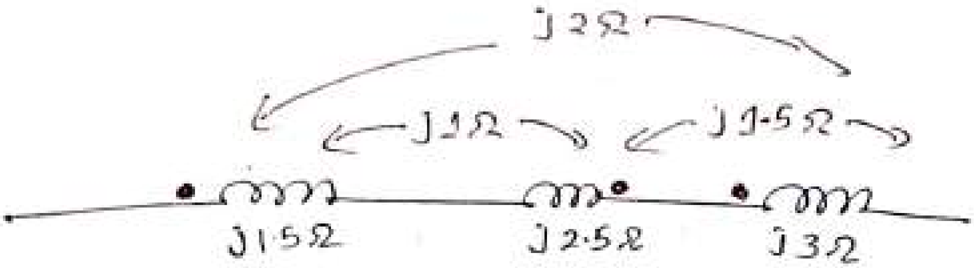
$$= j5 + j7$$

$$= j12\,\Omega \quad \text{(ANS)}$$

Q) Find the equivalent circuit & net inductance of iron cored coupled coil in series connection.



A ÷ equivalent circuit of iron cored coupled ckt



$$X_1 = j1.5 - j1\Omega + j2\Omega = j2.5\Omega$$

$$X_2 = j2.5 - j1\Omega - j1.5 = 0$$

$$X_3 = j3\Omega - j1.5 + j2\Omega = j3.5\Omega$$

Total reactance $= j6\Omega$

$$X = \omega L \Rightarrow L = \frac{X}{\omega} = \frac{6}{2\pi f} = \frac{6}{2 \times \pi \times 50}$$

$$= 0.019\,H \text{ (ANS)}$$

1) Voltage (V)

→ It is a force which can drive charges.
→ unit (volts).

$$\boxed{V = \frac{W}{Q} \left(\frac{\text{Joule}}{\text{columb}} \text{ or volt}\right)}$$

$$\boxed{V = \frac{dW}{dQ}}$$

2) current (i)

→ rate of flow of charge.
→ units → Amps.

$$\boxed{i = \frac{Q}{t} \quad \frac{\text{columb}}{\text{sec}} \text{ or Amp}}$$

$$\boxed{i = \frac{dq}{dt}}$$

3) Electric power (P)

→ rate of doing work → unit = watt or $\frac{\text{Joulse}}{\text{sec}}$ or J/s

$$\boxed{P = \frac{dW}{dt} = \frac{dW}{dq} \times \frac{dq}{dt} \\ P = V \times i}$$

$$\boxed{1 \text{ hp} = 746 \text{ Watt}}$$

4) Electrical Energy (E)

→ It is the capacity to do work electrically.

$$\boxed{E = \text{Power} \times \text{time}}$$

→ units → watt-sec or Joule

$$\boxed{1 \text{ unit of EE} = 1 \text{ KWhr}}$$

$$= 1000 \text{ W} \times 1 \text{ hr}$$

Circuit

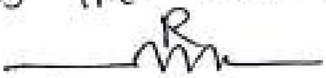
-> loop or mesh i.e. a closed path
-> current will flow through all elements.

Network

-> interconnection of circuits
-> may be closed or open

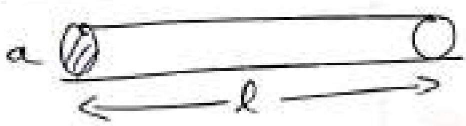
Resistance (R)

-> property of material or substance due to which it opposes the flow of electricity (i.e. electrons) in it.



-> units = (ohm) (Ω) or $\boxed{R = \frac{V}{A} \text{ or } \frac{\text{Volt}}{\text{Amp}}}$

basic formula



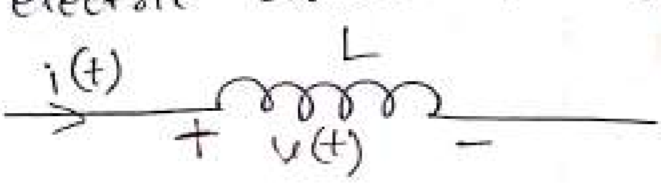
$\boxed{R = \rho \frac{l}{a}}$ Ω

ρ = resistivity of material (Ω-m unit)
l = length of conductor
a = area of cross section.

e.g. heater, bulb

Inductance (L)

-> property or tendency of an electrical conductor to oppose the change in electric current flowing through it.

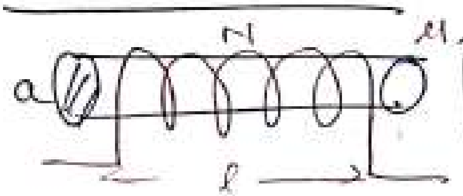


$V = L \frac{di}{dt}$ or $i = \frac{1}{L}\int v\,dt$

$L = \frac{V}{\left(\frac{di}{dt}\right)}$

unit $= \frac{\text{V-sec}}{\text{Amp}}$ or Henry

basic formula



$\boxed{L = \frac{\mu N^2 a}{l} \text{ H}}$

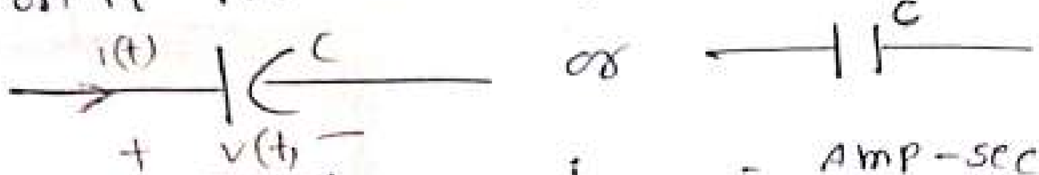
$\mu = \mu_0 \mu_r$ = permeability of core

N = No. of turns of coil
a = area of cross-section in m^2
ℓ = effective length of coil in m

ex: choke coil, Tx. line, filter

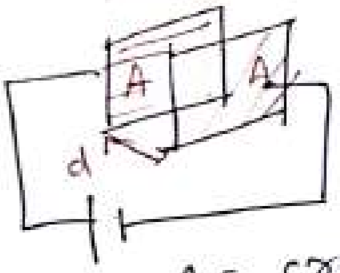
capacitor (C)

→ Property of an electric conductor or set of conductors, that is measured by the amount of separated electric charge that can be stored on it per unit change in electric potential



$$\boxed{i = C\frac{dV}{dt}} \qquad C = \frac{i}{(dV/dt)} = \frac{\text{Amp-sec}}{\text{Volt}} \text{ or Farad}$$

$$\boxed{V = \frac{1}{C}\int i\,dt}$$



$$\boxed{C = \frac{\epsilon A}{d}} \text{ Farad}$$

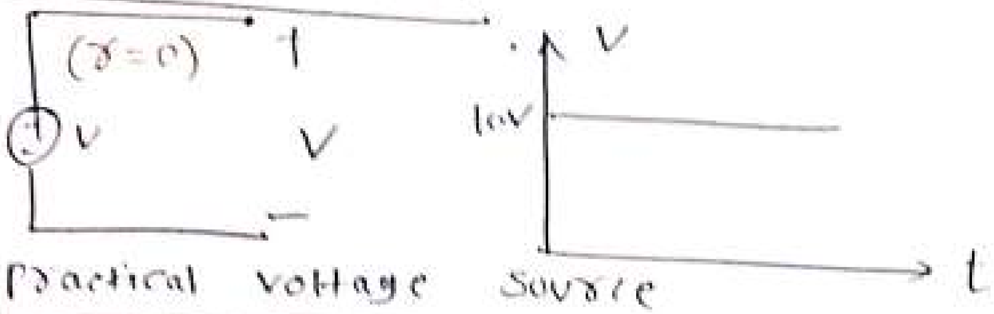
$\epsilon = \epsilon_0 \epsilon_r$ — permittivity of dielectric

$\epsilon_0 = 8.85 \times 10^{-12}$ F/m

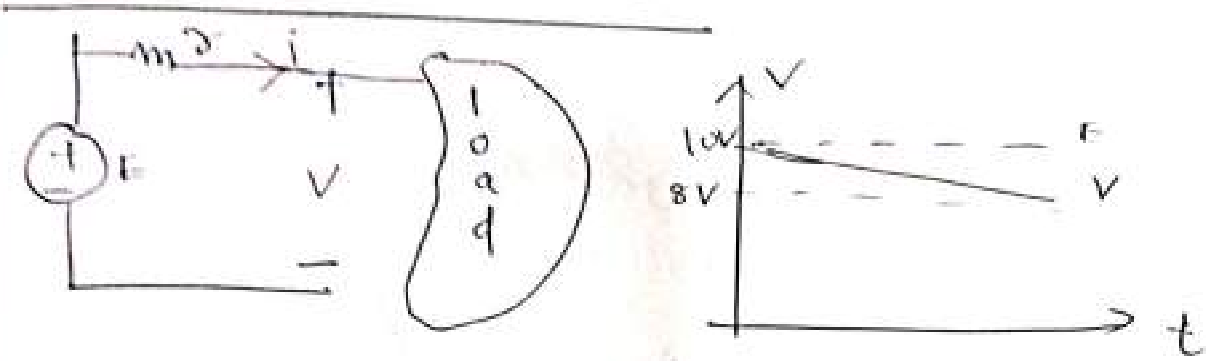
A = cross-sectional area of plates electrodes.
d = distance b/w the

Source
- Ideal
  - voltage source
    - independent
    - dependent
  - current source
    - Independent
    - dependent
- Practical
  - voltage source
    - Independent
    - dependent
  - current source
    - Independent
    - dependent

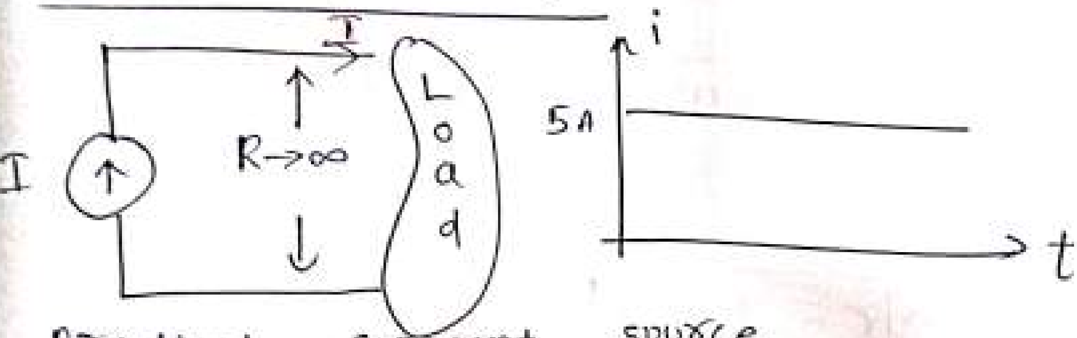
Ideal voltage source



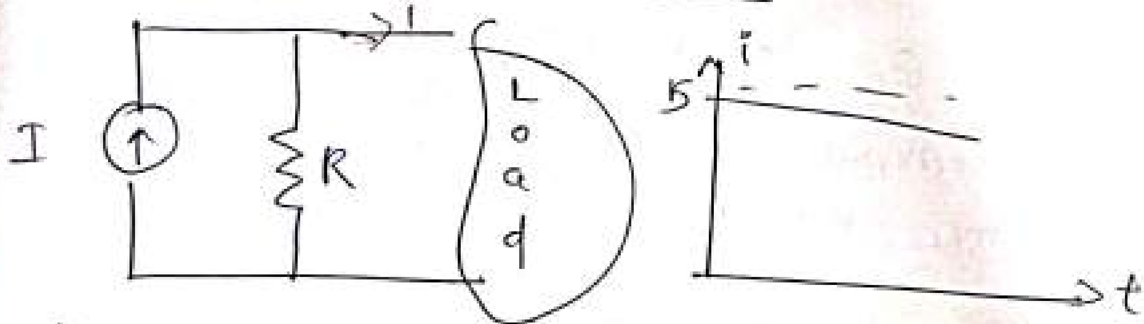
Practical voltage source



Ideal current source



Practical current source



KCL

→ based on law of conservation of charge.

$$\boxed{\sum i_{@\,Node} = 0}$$

KVL

→ based on law of conservation of energy.

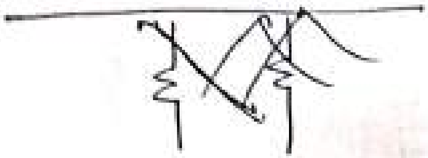
$$\boxed{\sum V_{@\,mesh} = 0}$$

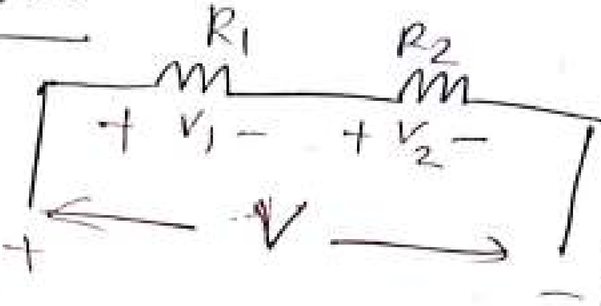
current division rule

$$I_1 = I \times \frac{R_2}{R_1 + R_2}$$

$$I_2 = I \times \frac{R_1}{R_1 + R_2}$$

voltage division rule

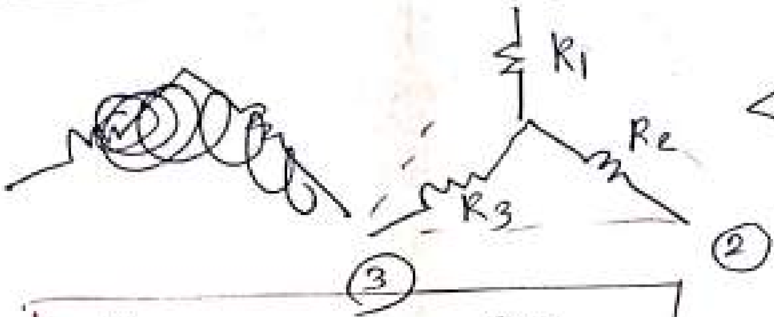


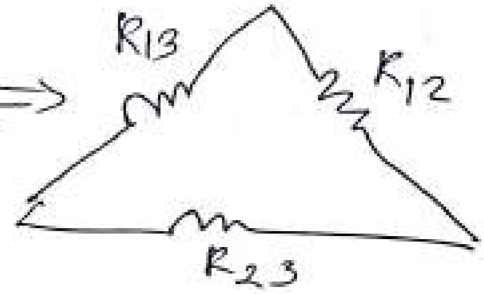


$$V_1 = V \times \frac{R_1}{R_1 + R_2}$$

$$V_2 = V \times \frac{R_2}{R_1 + R_2}$$

star to delta (Y − Δ)



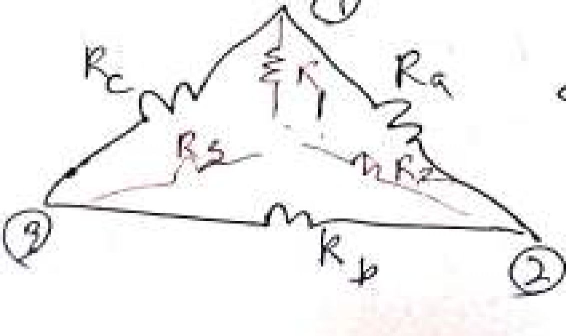


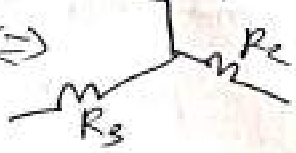
$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_{13} = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

delta to star (Δ − Y)





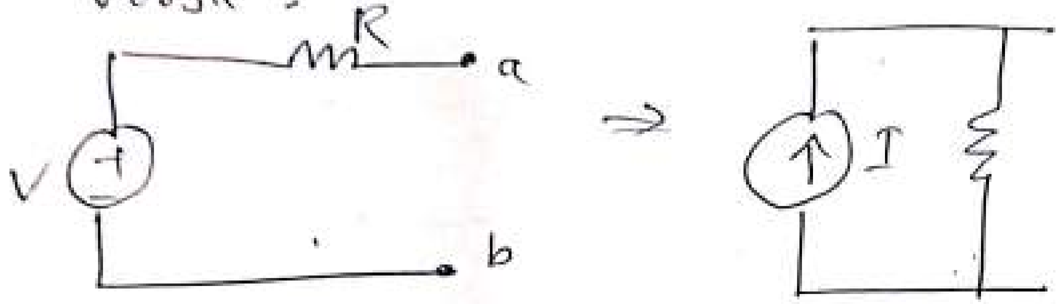
$$R_1 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_b}{R_a + R_b + R_c}$$

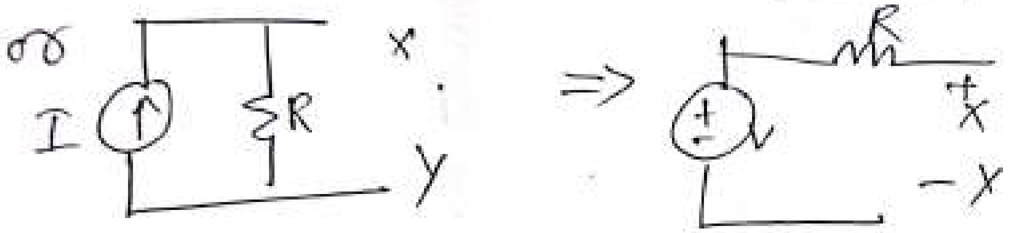
$$R_3 = \frac{R_b R_c}{R_a + R_b + R_c}$$

Source transformation technique

→ An ideal voltage source in series with resistance can be converted into an ideal current source in parallel with the same resistance & vice versa.



$\boxed{I = \frac{V}{R}}$



or $\boxed{V = IR}$

Types of Elements

ⓐ Active & passive

Active elements

→ Energy sources capable of delivering power or energy to some external device.

ex: generator, battery, BJT, op-amp, voltage source, current source,

Passive elements

→ elements that absorb electrical energy and convert it to some other forms of energy are said to be passive element.

ex. R, L, C, T/F, diode,

Active elements

→ The elements which supplies electrical energy to the circuit or provide power gain in the circuit are called Active elements.

→ ex ÷ Generator, Battery, op-amp, BJT,

Passive elements

→ The elements which consumes or absorbs or stores electrical energy in the circuit are called passive elements.

ex ÷ R, L, C, T/F, diode.

Bilateral elements

→ Bilateral elements are those elements that allow the current in both direction & offer same impedance in both direction of current flow.

→ ex ÷ Resistor(R), inductors(L) & capacitors(C)

$V = 3 \times 2 = 6\,V$

$P = 6 \times 3 = 18\,W$

$V = 3 \times 2 = 6\,V$

$P = 6 \times 3 = 18\,W$

Unilateral elements

→ are those who allow current to flow in one direction & stops current in another direction.

→ offers different resistances in both directn

→ ex ; Diode, BJT, MOSFET

diode

forward biased
→ current will flow
→ R = 0Ω

→ reverse biased
→ I = 0
→ R = ∞ Ω

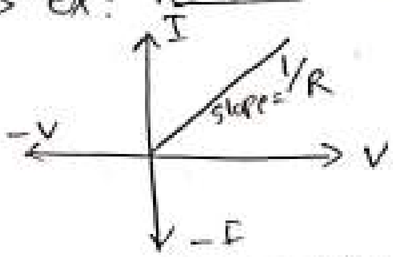
Linear elements

→ The relation b/w voltage & current is a linear function.

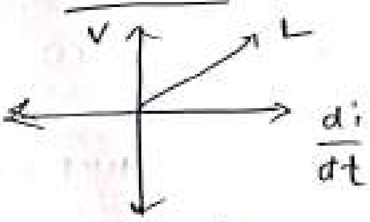
(or)
→ The element that characterstics are straight line and passing through the origin is known as linear element otherwise non-linear.

(or)
→ An element is said to be linear, if its V-I characteristics is at all times a straight line passing through origin.

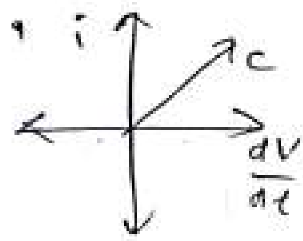
→ ex: Resistor, inductor, capacitor



$V = IR$, $I = \left(\frac{1}{R}\right)V$



$V = L\frac{di}{dt}$



Non linear element . diode, transistors

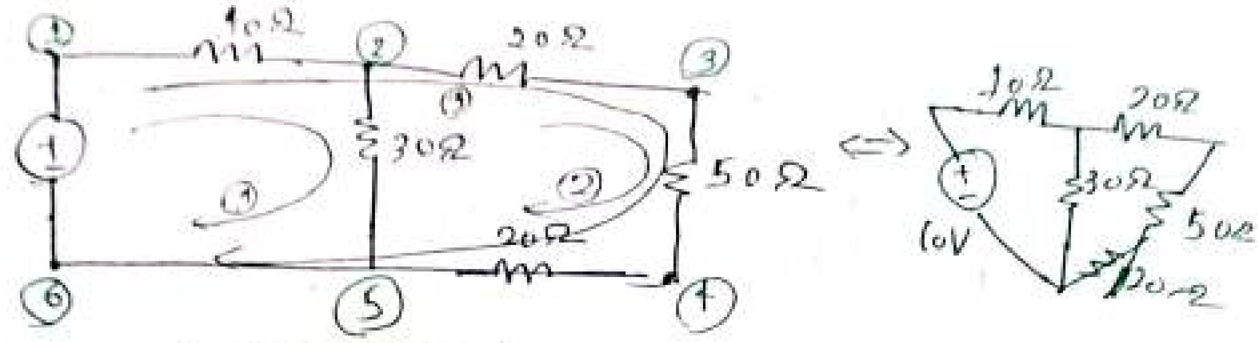
Node : It is a junction in a circuit where two or more circuit elements are joined together.

branch : It is that part of a network which lies b/w two junctions.

loop : It is a closed path in a circuit in which no element or node is encountered more than once.

Mesh: It is a loop that contains no other loop within it.

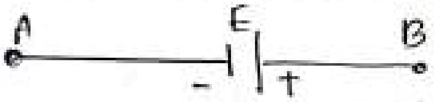
ex



No. of branches = 7
No. of Nodes = 6
No. of loops = 3
No. of Mesh = 2

Determination of Voltage sign in KVL

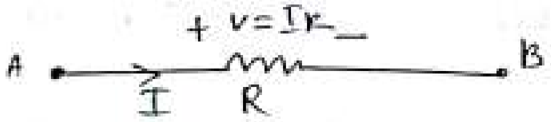
a) sign of battery or E.M.F

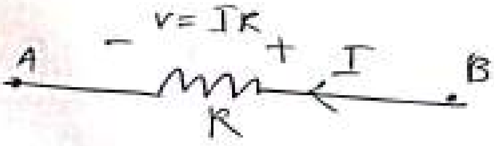


→ motion
rise in Voltage
$+E$

→ motion
drop in voltage
$-E$

b) sign of IR drop





→ motion
drop in voltage
$-V = -IR$

→ motion
rise in voltage
$+V = +IR$

Planar circuit / Network

A circuit is said to be planar, if it can be drawn on a plane surface without cross-over.

Cramer's rule

$a_1x + b_1y = c_1$ $\Rightarrow \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

$a_2x + b_2y = c_2$

$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ $\quad \Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1b_2 - c_2b_1$

$= a_1b_2 - a_2b_1$ $\quad \Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = a_1c_2 - a_2c_1$

$$x = \frac{\Delta_x}{\Delta} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}$$

$$y = \frac{\Delta_y}{\Delta} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$$

similary for '3' variables it can be done.

$\boxed{\text{Mesh analysis}}$ → (KVL + ohm's law)

→ applicable only for planar Networks.

Steps For solving N/w using mesh Analysis

Step 1: simplify the given circuit, if possible by replacing the current sources by the corresponding equivalent voltage sources

Step 2: Idientify the meshes as 1, 2, 3 . . .

Step 3: Designate the mesh currents as I_1, I_2, I_3. mesh currents are assumed either in clockwise or anti-clockwise direction. (usually clockwise)

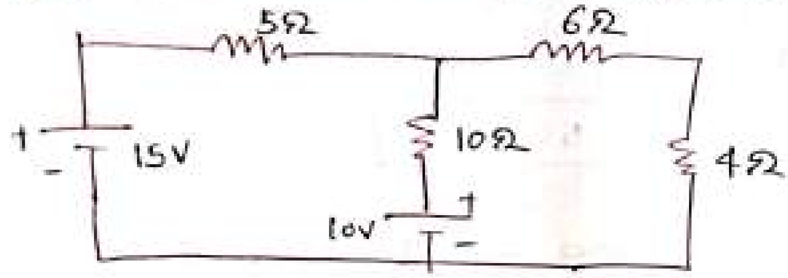
Step-4: Using Kirchoff's voltage law (KVL) write

Mesh eqn for all the meshes.

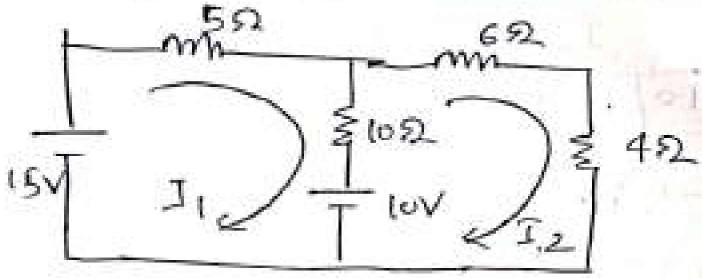
<u>Step-5</u> Solve the mesh eqn using Cramer's rule or any other suitable method.

<u>Step-6</u> Go for circuit analysis i.e. finding current, voltage & power dissipation against an element in the given circuit.

Q) Using mesh analysis, Find current through 10Ω & 4Ω resistance in the circuit ?

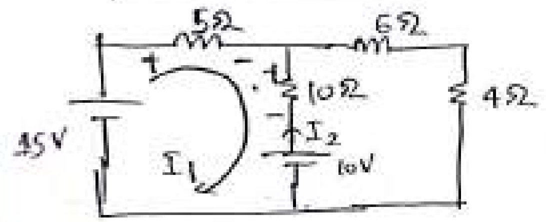


A:



Identify '2' No. of mesh's & assign clockwise current directions as I_1 & I_2.

<u>For mesh-1</u>



$15 - 5I_1 - 10(I_1 - I_2) - 10 = 0$

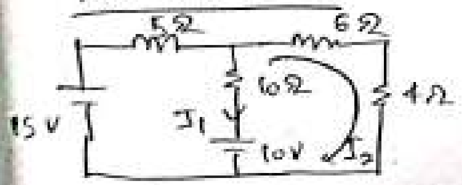
$15 - 5I_1 - 10I_1 + 10I_2 - 10 = 0$

$\Rightarrow -15I_1 + 10I_2 + 5 = 0$

$\Rightarrow 15I_1 - 10I_2 = 5$

$\Rightarrow \boxed{3I_1 - 2I_2 = 1 \quad \ldots (i)}$

<u>For mesh-2</u>



$10 - 10(I_2 - I_1) - 6I_2 - 4I_2 = 0$

$\Rightarrow 10 - 10I_2 + 10I_1 - 10I_2 = 0$

$\Rightarrow 10I_1 - 20I_2 = -10$

$\Rightarrow \boxed{I_1 - 2I_2 = -1} \quad \ldots (ii)$

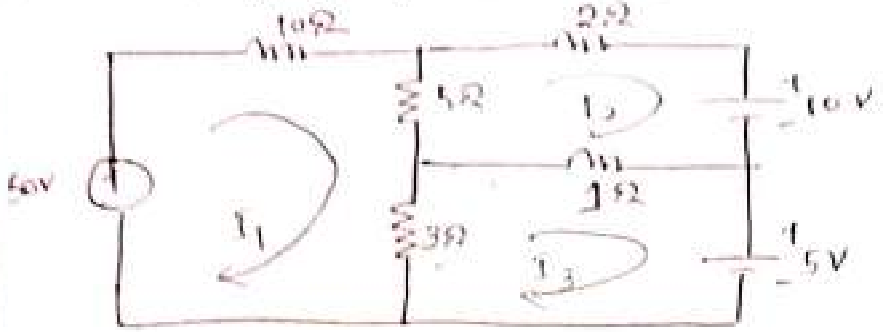
eqn (i) − eqn (ii)

$2I_1 = 2$, $\boxed{I_1 = 1\,\text{Amp}}$, put I_1 in eqn

$1 - 2I_2 = -1 \Rightarrow \boxed{I_2 = 1\,\text{Amp}}$

current through $10\Omega = I_1 - I_2 = 0\,\text{Amp}$, $I_{4\Omega} = I_2 = 1\,\text{Amp}$ (Ans)

Q) Find I_1, I_2, I_3 by using mesh eqn :



A: For mesh ①

$$50 - 10I_1 - 5(I_1 - I_2) - 3(I_1 - I_3) = 0$$

$$\Rightarrow 50 - 10I_1 - 5I_1 + 5I_2 - 3I_1 + 3I_3 = 0$$

$$\Rightarrow 50 - 18I_1 + 5I_2 + 3I_3 = 0$$

$$\Rightarrow \boxed{18I_1 - 5I_2 - 3I_3 = 50} \quad \cdots (i)$$

m-2

$$-5(I_2 - I_1) - 2I_2 - 10 - 1(I_2 - I_3) = 0$$

$$\Rightarrow -5I_2 + 5I_1 - 2I_2 - 10 - I_2 + I_3 = 0$$

$$\Rightarrow \boxed{5I_1 - 8I_2 + I_3 = 10} \quad \cdots (ii)$$

m-3

$$-3(I_3 - I_1) - (I_3 - I_2) - 5 = 0$$

$$\Rightarrow -3I_3 + 3I_1 - I_3 + I_2 = 5$$

$$\Rightarrow \boxed{3I_1 + I_2 - 4I_3 = 5} \quad \cdots (iii)$$

According to cramer's rule

$$I_1 = \frac{\begin{vmatrix} 50 & -5 & -3 \\ 10 & -8 & 1 \\ 5 & 1 & -4 \end{vmatrix}}{\begin{vmatrix} 18 & -5 & -3 \\ 5 & -8 & 1 \\ 3 & 1 & -4 \end{vmatrix}} = \frac{1176}{356} \text{ (Ans)}$$

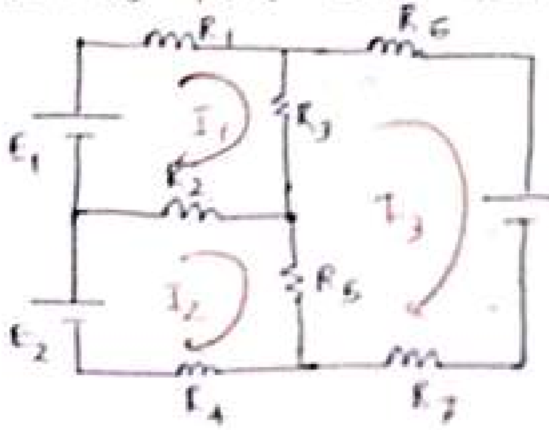
$$I_3 = \frac{\begin{vmatrix} 18 & -5 & 50 \\ 5 & -8 & 10 \\ 3 & 1 & 5 \end{vmatrix}}{\begin{vmatrix} 18 & -5 & -3 \\ 5 & -8 & 1 \\ 3 & 1 & -4 \end{vmatrix}} = \frac{525}{356} \text{ (Ans)}$$

$$I_2 = \frac{\begin{vmatrix} 18 & 50 & -3 \\ 5 & 10 & 1 \\ 3 & 5 & 4 \end{vmatrix}}{\begin{vmatrix} 18 & -5 & -3 \\ 5 & -8 & 1 \\ 3 & 1 & -4 \end{vmatrix}} = \frac{355}{356} \text{ (Ans)}$$

Mesh eq^n by inspection method

The mesh eq^n for a general planar N/W can be written by inspection without going through the detailed steps.

→ Let us take an example



Applying mesh analysis

(1) $E_1 = (R_1+R_2+R_3)I_1 - I_2R_2 - I_3R_3$

(2) $E_2 = -R_2I_1 + (R_2+R_4+R_5)I_2 - R_5I_3$

(3) $E_3 = -R_3I_1 - R_5I_2 + (R_3+R_5+R_6+R_7)I_3$

or $E_1 = R_{11}I_1 - R_{12}I_2 - I_3R_{13}$

$E_2 = -R_{21}I_1 + R_{22}I_2 - R_{23}I_3$

$E_3 = -R_{31}I_1 - R_{32}I_2 + R_{33}I_3$

where $R_{11} = (R_1+R_2+R_3)$
= self resistance of mesh (1)

R_{22} = self resistance of mesh (2)
i.e. sum of all resistances in mesh (2)
$= (R_2+R_4+R_5)$

R_{33} = self resistance of mesh (3) $= (R_3+R_5+R_6+R_7)$

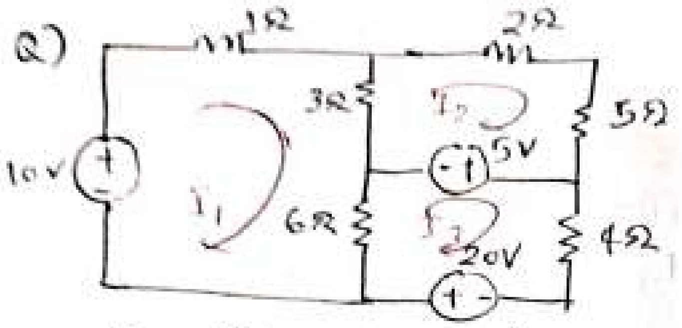
$R_{12} = R_{21} = -$[sum of all the resistances common to mesh (i) & mesh (ii)]

$R_{23} = R_{32} = -$[Sum of all the resistances common to mesh (ii) & mesh (iii)]
or mutual resistance b/w mesh (i) & mesh (ii)

$R_{13} = R_{31} = -$[sum of all the resistances common to mesh (i) & mesh (iii)]

Note If we assume each mesh current as clockwise
i) all self-resistances will always be positive
ii) all mutual resistances will always be Negative.

E_1 = represents the algebraic sum of the voltages of all the voltage sources acting around mesh (1)
(similarly E_2 & E_3)

Q) 

write the mesh eqn for the circuit ?

A: No. of mesh = 3

$R_{11} = (1Ω + 3Ω + 6Ω) = 10Ω$ $\quad R_{12} = R_{21} = -3Ω$

$R_{22} = (3Ω + 2Ω + 5Ω) = 10Ω$ $\quad R_{13} = R_{31} = -6Ω$

$R_{33} = (6Ω + 4Ω) = 10Ω$ $\quad R_{23} = R_{32} = -0Ω = 0Ω$

$V_1 = 10,\ V_2 = -5V,\ V_3 = 5 + 20 = 25V$

eqn

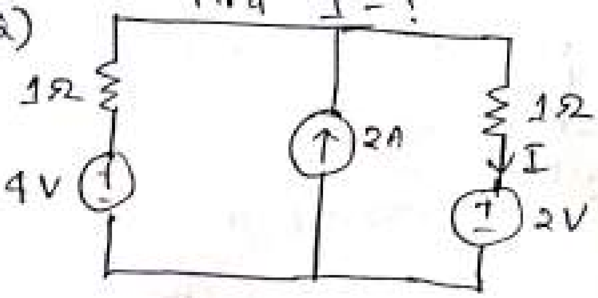
$$\boxed{\begin{aligned} 10I_1 - 3I_2 - 6I_3 &= 10\ V \\ -3I_1 + 10I_2 &= -5\ V \\ -6I_1 + 10I_3 &= 25\ V \end{aligned}}$$

(ANS).

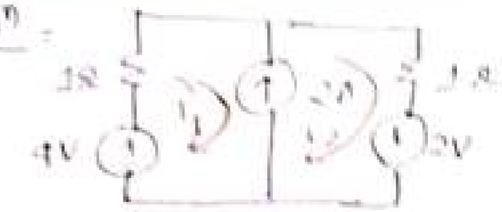
Supermesh Analysis

Supermesh: when a current source is present b/w two meshes, we remove the branch having the current source & then the remaining loop is known as supermesh.

Q) Find I = ?



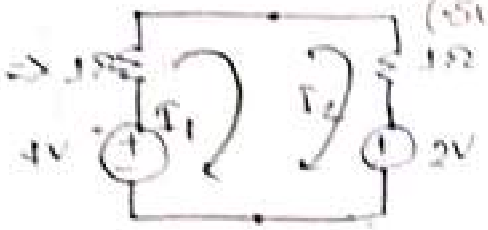
Soln.



i) No. of mesh = 2

ii) Remove the branch having current source.

(Supermesh)



Apply KVL in supermesh

$$4 - I_1 - I_2 - 2 = 0$$

$$\boxed{I_1 + I_2 = 2} \cdot (i)$$

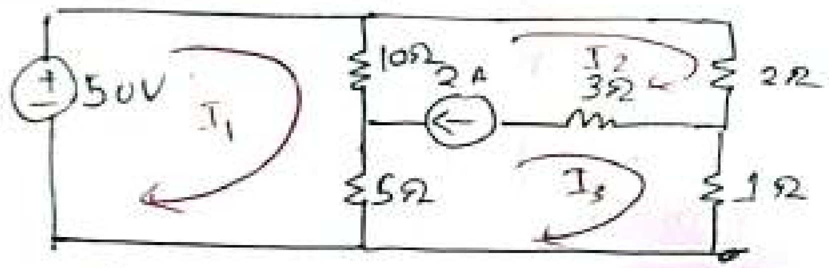
→ I_2 & 2A are flowing in same direction, so we can write $\boxed{I_2 - I_1 = 2}$. (ii)

eqn (i) + eqn (ii) $2I_2 = 4$, $I_2 = 2$ Amp

$I_1 = 2 - 2 = 0$ Amp

$$\boxed{I_2 = I = 2 \text{ Amp}}$$

Q)



current in 5Ω resistor = ?

A: Applying KVL to mesh (1)

$$\boxed{15I_1 - 10I_2 - 5I_3 = 50} \cdot (1)$$

→ mesh (ii) & mesh (iii) shares a common current source and it forms a supermesh.

→ writing eqn for supermesh

I_2 & 2A flow in same direction so $\boxed{I_2 - I_3 = 2}$. (ii)

→ Assume 2A current source is not there.

Applying KVL to supermesh

$\cancel{-10I_2 - 2I_2 - I_3 - 5I_3 = 0}$

$\cancel{-12I_2 - 6I_3 = 0}$

$$-10(I_2 - I_1) - 2I_2 - I_3 - 5(I_3 - I_1) = 0$$

$\boxed{-15I_1 + 12I_2 + 6I_3 = 0}$ ... (iii)

$I_1 = 19.99\ \text{amp}$

$I_2 = 17.33\ \text{amp}$

$I_3 = 15.33\ \text{amp}$

$$I_{5\Omega} = I_1 - I_3 = 19.99 - 15.33 = 4.66\ \text{amp}\ \text{(ANS)}$$

Note

No. of linearly independent mesh eqⁿ m = branches − (Nodes − 1)

$$\boxed{m = b - (N - 1)}$$

Nodal Analysis (KCL + ohm's law)

i) Applicable for both planar & non-planar N/ws.

ii) No. of eqns required to solve an electrical N/w is $\boxed{e = N - 1}$ N = No. of Nodes

Procedure of Nodal Analysis

Step-1 : Identify the principal nodes and choose one of them as reference node. We will treat that reference node as the ground.

Step-2 : Label the node voltages w.r.t. ground from all the principal nodes except the reference node. (datum node)

Step-3 write Nodal equations at all the principal nodes except the reference node. Nodal eqⁿ is obtained by applying KCL first & then ohm's law.

Step-4 : Solve the Nodal equations obtained in step-3 in order to get the Node Voltages.

(1.1)

Node : The common point where two or more elements are connected.

Simple Node : the common point where two elements are connected. (current division doesn't happen)

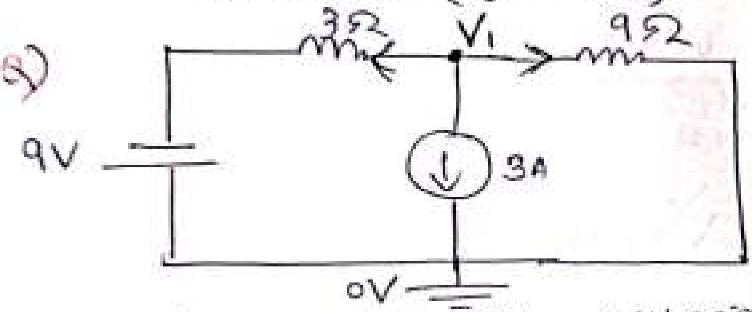
Principal node : the common point where more than two elements are connected. (current division occurs)

$$\boxed{I = \frac{V_1 - V_2}{R_1}}$$

→ Assume one principal Node as reference or datum node.

Reference (ground, V = 0)

Q) Find node voltage V_1 = ?



A :- → Always assume outgoing current at each node.
→ current always flows from high voltage to low voltage.

Apply KCL at Node V_1

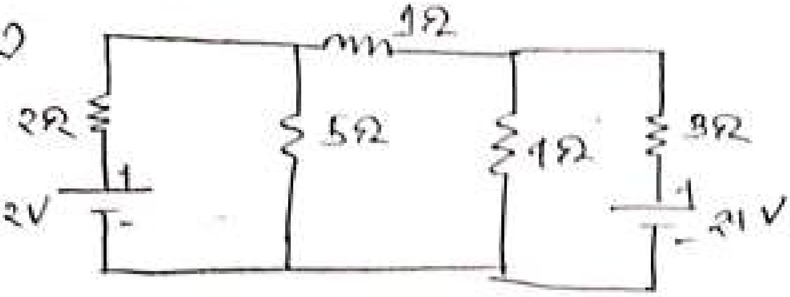
Assume incoming current source as −ve and outgoing current source as +ve

$$\frac{V_1 + 9}{3} + \frac{V_1 - 0}{9} + 3 = 0$$

$\Rightarrow 4V_1 = 54$

$\Rightarrow \boxed{V_1 = -13.5 \text{ volt}}$

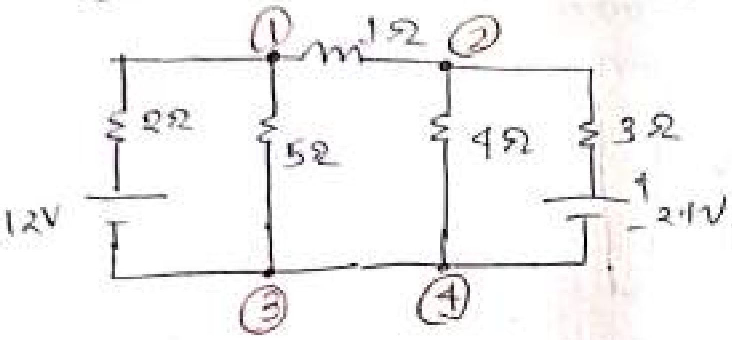
1) Q)



Calculate current flowing through 1Ω resistor by Nodal Analysis?

A: (1) Identify all the principal nodes

2)

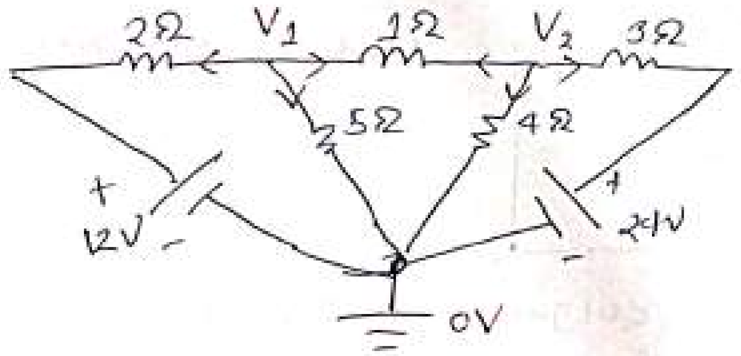


(2) Node '3' & '4' are taken as one node as there is no parameter b/w '3' & '4'.

So this node is called reference node or datum node.

(3) Assume voltage of Node (1) as V_1 & (2) as V_2.

3



4 (4) KCL at Node (1)

$$\frac{V_1-12}{2}+\frac{V_1-0}{5}+\frac{V_1-V_2}{1}=0$$

$$\Rightarrow \frac{5(V_1-12)+2V_1+10V_1-10V_2}{10}=0$$

$$\Rightarrow 5V_1-60+2V_1+10V_1-10V_2=0$$

$$\Rightarrow \boxed{17V_1-10V_2=60 \quad \cdots (1)}$$

KCL at node (2)

$$\frac{V_2-24}{3}+\frac{V_2-0}{4}+\frac{V_2-V_1}{1}=0$$

$$\Rightarrow \frac{4(V_2-24)+3V_2+12V_2-12V_1}{12}=0$$

$\Rightarrow 4V_2 - 96 + 3V_2 + 12V_2 - 12V_1 = 0$

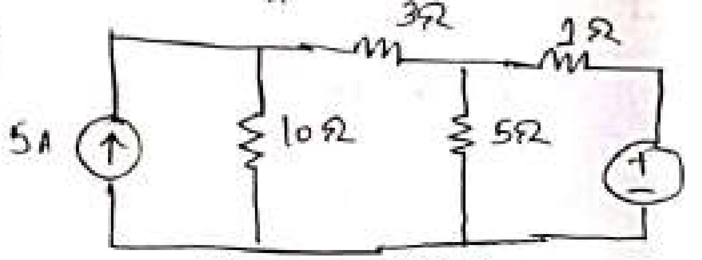
$\Rightarrow \boxed{-12V_1 + 19V_2 = 96}$ ...(ii)

Solving eq^n (i) & (ii) $V_1 = \frac{2100}{203}$ volts, $V_2 = \frac{2352}{203}$ volt

$$I_{3\Omega} = \frac{V_1 - V_2}{1} = \frac{2100 - 2352}{203} = -1.24 \text{ Amps.}$$

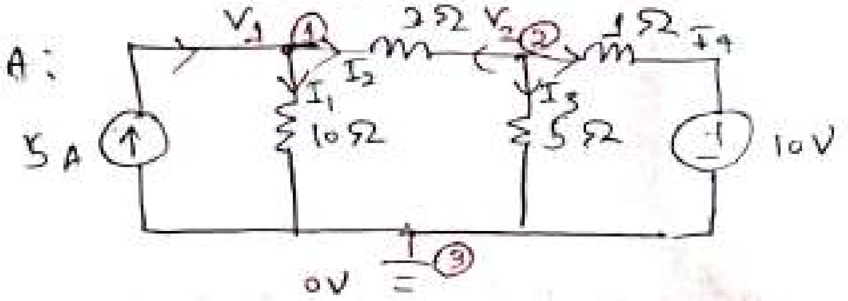
∴ The −ve sign indicates the current is ~~from~~ flowing from V_2 to V_1.

Q)



write the Node voltage eq^n & find the currents in each branch.

A:



KCL at Node '1'

$$\frac{V_1 - V_2}{3} + \frac{V_1 - 0}{10} - 5 = 0 \Rightarrow \frac{10(V_1 - V_2) + 3V_1}{30} = 5$$

$\Rightarrow 10V_1 - 10V_2 + 3V_1 = 150$

$\Rightarrow \boxed{13V_1 - 10V_2 = 150}$ (i)

KCL at Node '2'

$$\frac{V_2 - 10}{1} + \frac{V_2 - V_1}{3} + \frac{V_2 - 0}{5} = 0 \Rightarrow \frac{15(V_2 - 10) + 5(V_2 - V_1) + 3V_2}{15} = 0$$

$\Rightarrow 15V_2 - 150 + 5V_2 - 5V_1 + 3V_2 = 0 \Rightarrow \boxed{-5V_1 + 23V_2 = 150}$ (ii)

Solving eq^n (i) & (ii)

$V_1 = 19.85V$, $V_2 = 10.9\ V$

$I_1 = \frac{V_1 - 0}{10} = \frac{19.85}{10} = 1.985$ Amp

$I_2 = \frac{V_1 - V_2}{3} = \frac{19.85 - 10.9}{3} = 2.98$ Amp

$I_3 = \frac{V_2 - 0}{5} = \frac{10.9}{5} = 2.18$ Amp

, $I_4 = \frac{V_2 - 10}{1}$

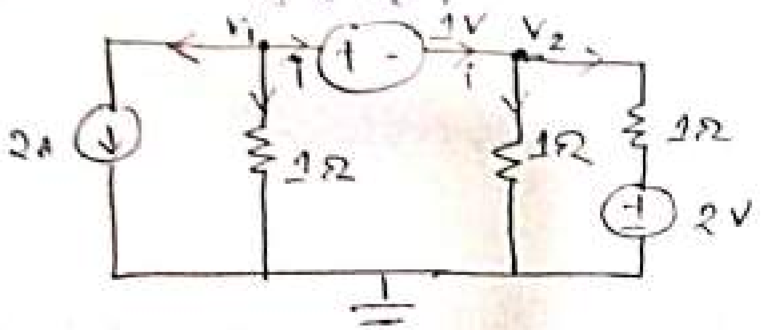
$= 10.9 - 10$

$= 0.9$ Amp

Super Node

When the voltage source is connected b/w two non-reference nodes.

ex: Find V_1 & V_2.



No. of Nodes = 3

A:

At Node '1'

$$2 + \frac{V_1 - 0}{1} + i = 0$$

$$\Rightarrow \boxed{2 + V_1 + i = 0} \ldots (i)$$

At node '2'

$$\frac{V_2 - 2}{1} + \frac{V_2 - 0}{1} = i$$

$$\Rightarrow 2V_2 - 2 = i$$

$$\Rightarrow \boxed{2V_2 - i - 2 = 0} \ldots (ii)$$

eq^n (i) + eq^n (ii)

$$\boxed{V_1 + 2V_2 = 0} \ldots (iii)$$

Apply KVL in supernode i.e. from V_1 to V_2

$$V_1 - 1 = V_2 = 0$$

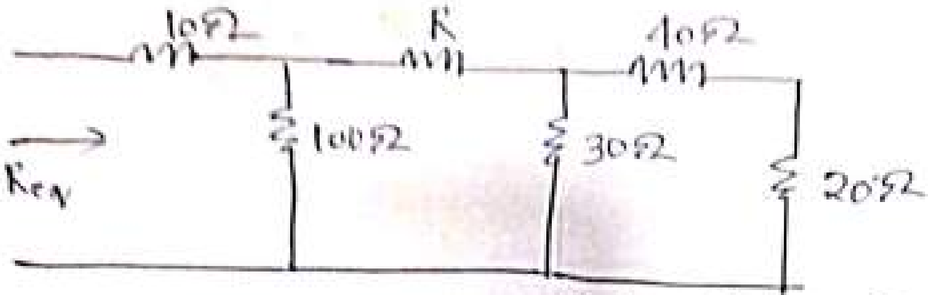
$$\boxed{V_1 - V_2 = 1} \ldots (iv)$$

eq^n (iii) − eq^n (iv) $3V_2 = -1$ $\boxed{V_2 = -1/3}$

$$\boxed{V_1 = 1 + V_2 = 1 + (-1/3) = 2/3 \text{ volt}} \text{ (Ans)}$$

Assignment

1)

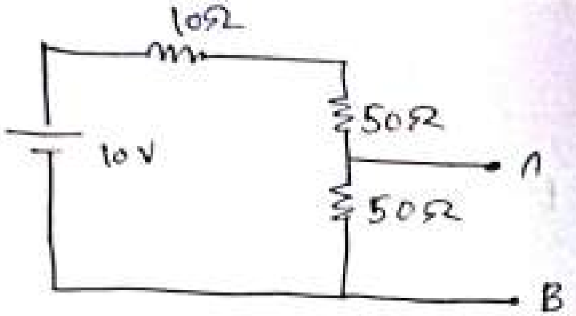


In the above Network a) Let R = 80Ω, Find R_{eq} = ?

b) Find R if R_{eq} = 80Ω

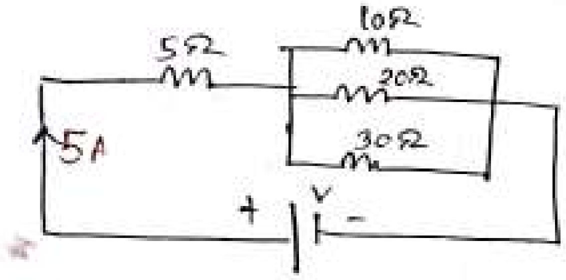
c) Find R if R = R_{eq}

2)



Find value of V_{AB} = ?

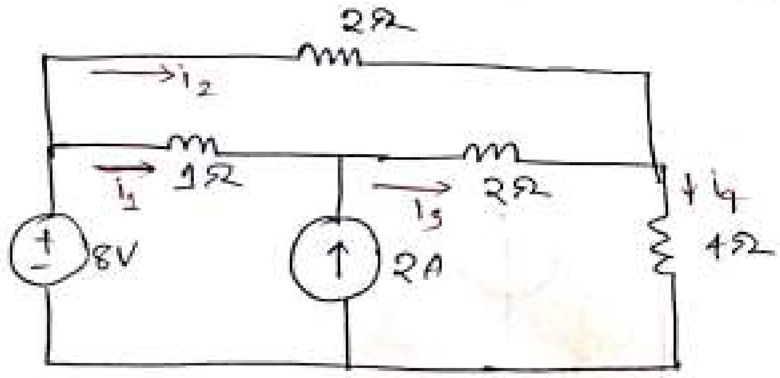
3)



The current in 5Ω resistance of the circuit is 5A. i) Find the current in 10Ω resistor

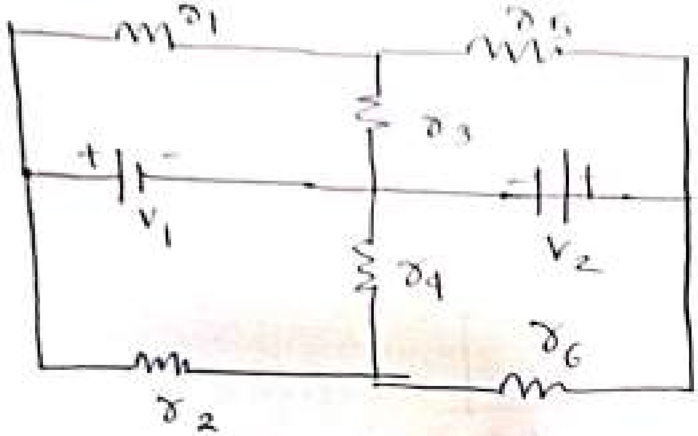
ii) calculate power consumed by 5Ω resistor ?

4)

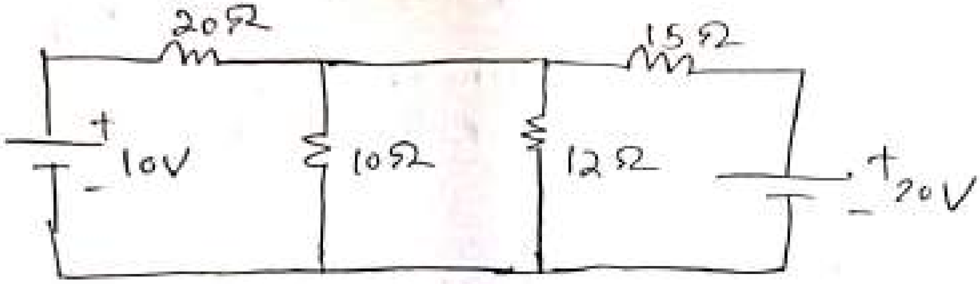


Find i_1, i_2, i_3, i_4 by using Nodal Analysis.

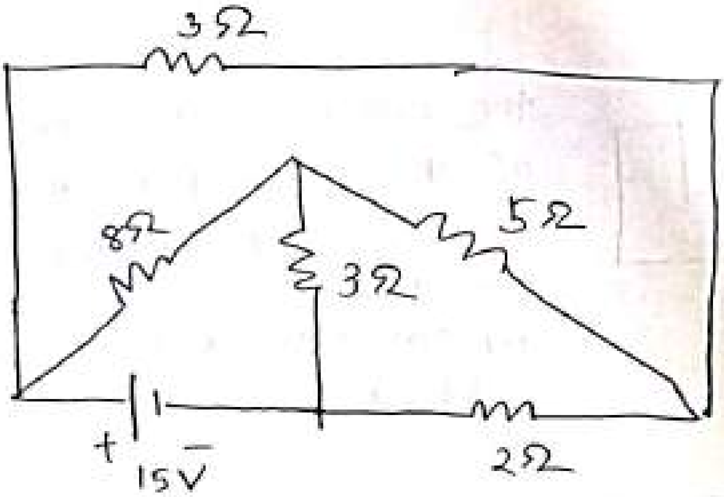
(*) star to delta (Y to Δ) transformation

5) 

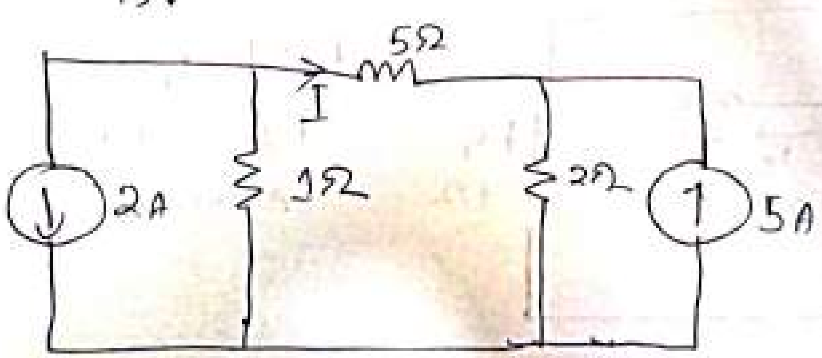
For this circuit write down mesh equations ?

6) 

Using Nodal Analysis Find the current through 15 Ω resistor !

7) 

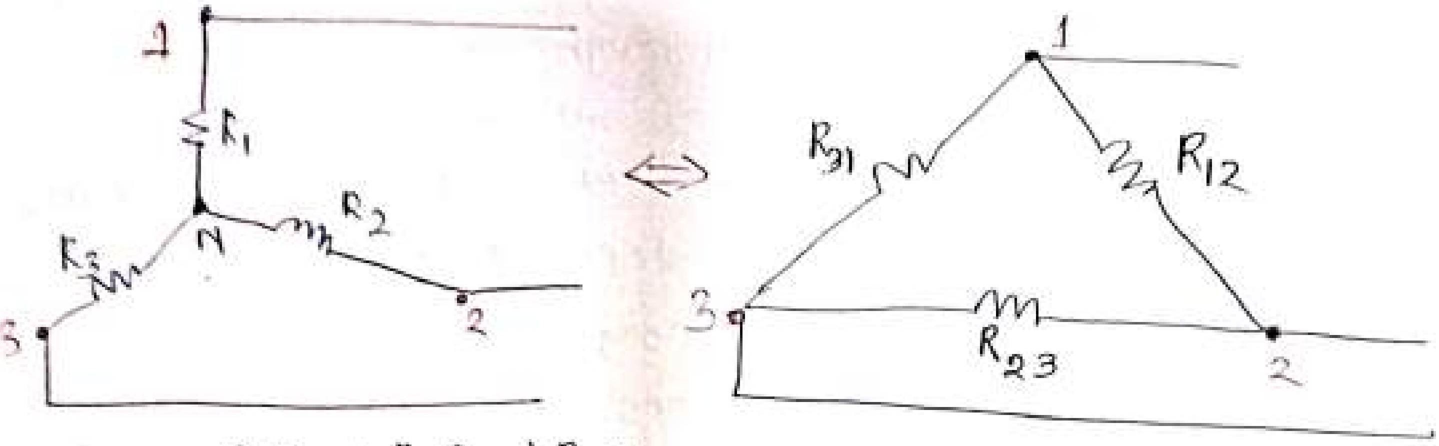
write mesh equations for the above circuit

8) 

Find $I_{5\Omega}$ by i) mesh analysis
ii) Nodal Analysis .

(1). NETWORK THEOREMS

* star to delta (Y to Δ) transformation



$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

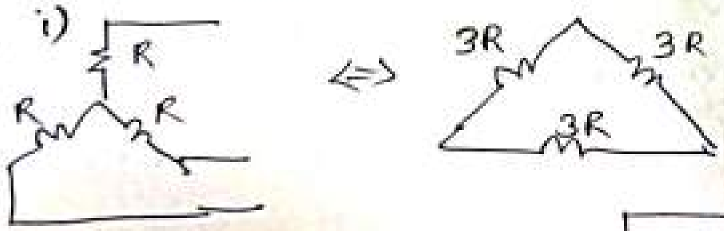
$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = R_3 + R_1 + \frac{R_3R_1}{R_2}$$

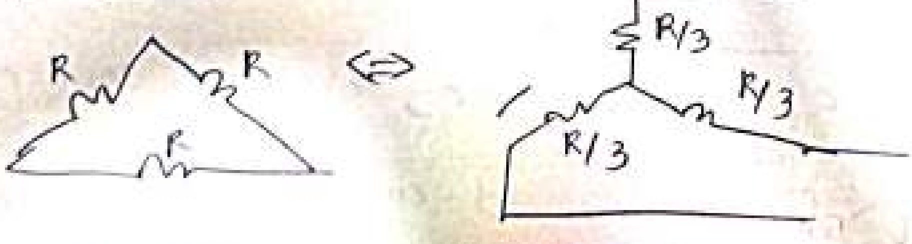
* delta to star conversion (Δ to Y)

$$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}, \quad R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}, \quad R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$$

Note i)



ii)



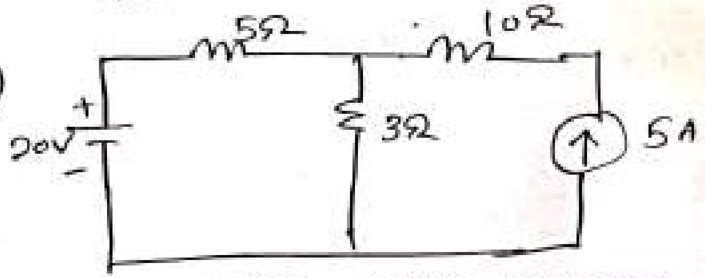
→ used for solving equivalent resistance in complex N/ws.

Superposition theorem

→ **Statement** :- In a linear, bilateral, active Network containing more than one source ~~or~~ ~~more~~, the current which flows through any circuit element is the algebraic sum of all the currents which would flow through that element if each ~~one of~~ source was considered separately and all other sources are ~~are~~ de-energised & replaced by their internal resistances.

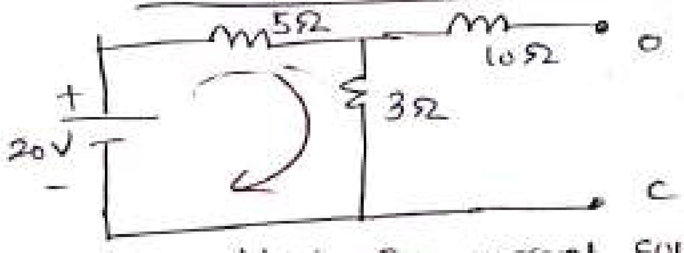
→ This theorem is only applicable for linear N/ws.

→ While considering the effect of indivisual sources, other ideal voltage sources are replaced by short circuit & current sources are replaced by open circuit.

Q) 

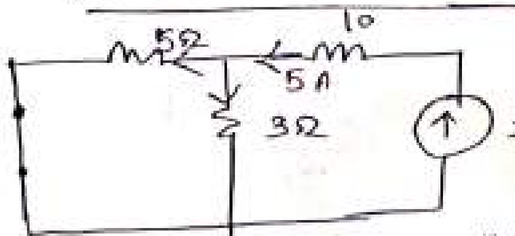
Calculate current passing through 3Ω resistor by SPT ?

A- Consider 20V source, current source deactivated. i.e. open circuited.



$$I_{3\Omega} = \frac{20}{5+3} = \frac{20}{8} = I_1$$
$$= 2.5\ Amp$$

Consider 5A current source, voltage source deactived. i.e. short circuited.

SC 

by current division rule

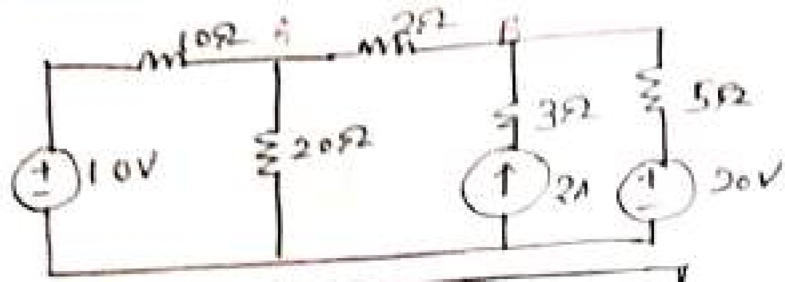
$$5A\quad I_2 = I_{3\Omega} = 5 \times \frac{5}{8} = \frac{25}{8} = 3.125\ Amp$$

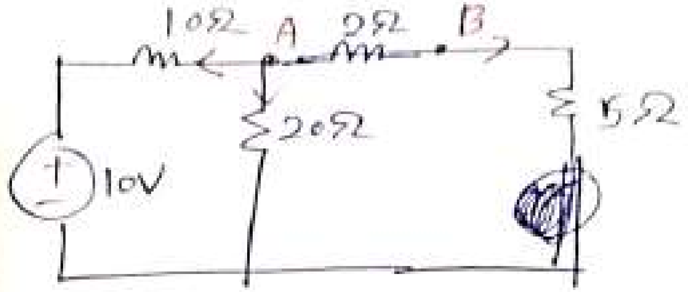
The total current passing through 3Ω resistor
$= I = I_1 + I_2 = 2.5 + 3.125 = 5.625\ (Amp)$

(3)

Step-5 Find current through 'r'

Q) Find the voltage across the 2Ω resistor by using SPT ?





Consider 10V source

Short circuit = 20V
open circuit = 2A

Put Nodal Analysis at 'A'

$$\frac{V_A-10}{10}+\frac{V_A}{20}+\frac{V_A}{7}=0$$

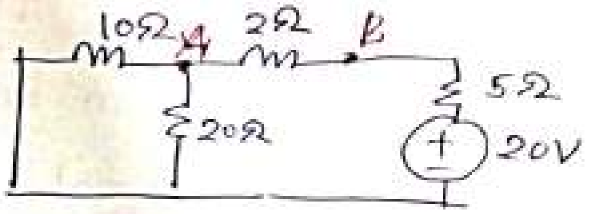
$$\Rightarrow \frac{14V_A-140+7V_A+20V_A}{140}=0$$

$$\Rightarrow 41V_A = 140 \quad \Rightarrow V_A=\frac{140}{41}$$

Voltage in '2Ω' by putting voltage divison rule

$$V_1=V_{2\Omega}= V_A \times \frac{2}{2+5} = \frac{140}{41}\times\frac{2}{7} = \frac{40}{41} = 0.975 \text{ volt}$$

Consider 20V source



Short circuit = 10V
open circuit = 2A

Nodal Analysis at 'A'

$$\frac{V_A-20}{7}+\frac{V_A}{20}+\frac{V_A}{10}=0$$

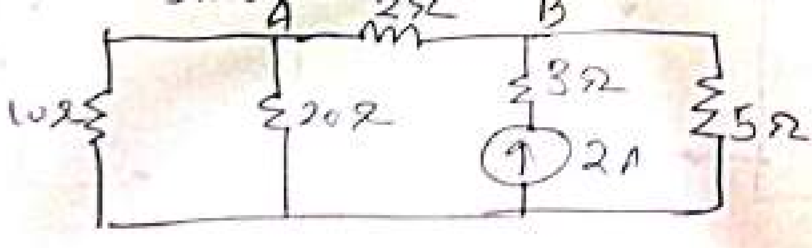
$$\Rightarrow \frac{20V_A-400+7V_A+14V_A}{140}=0$$

$$\Rightarrow 41V_A=400 \Rightarrow V_A=\frac{400}{41} \text{ volt.}$$

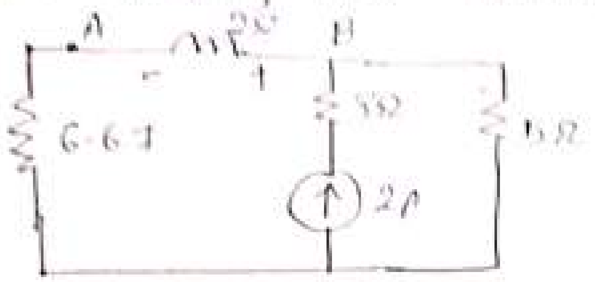
$$V_2 = V_{2\Omega} = \left(\frac{V_A-20}{7}\right)\times 2 = \left(\frac{400/41-20}{7}\right)\times 2 = -2.92 \text{ volt}$$

Consider 2A current source

Short circuit = 10V & 20V.



~~Simplify~~ simplify the circuit



$$I_{2\Omega} = 2 \times \frac{5}{13.67} = \frac{10}{13.67} = 0.73 \text{ Amp}$$

V_3 voltage in $2\Omega = I_{2\Omega} \times 2 = 0.73 \times 2 = 1.46$ volt.

The algebraic sum of all these voltage gives the total voltage across the 2Ω resistor in the N/W $= V = V_1 + V_2 + V_3$

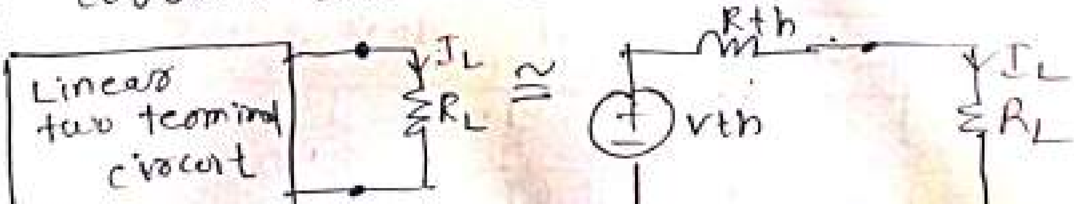
$$= 0.97 - 2.92 - 1.46$$
$$= -3.41 V$$

→ -ve sign indicates voltage at 'A' is -ve.

THEVENIN'S THEOREM

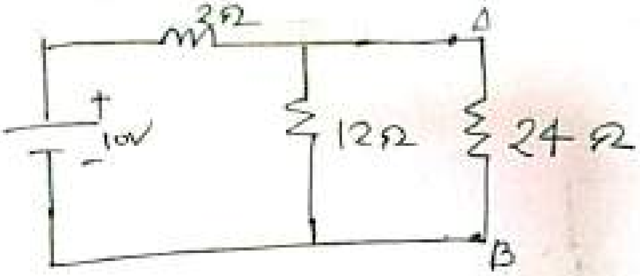
Statement: In any linear, active, bilateral ~~network~~ two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th}, where V_{th} is the open circuit voltage at the load terminals & R_{th} is the input or equivalent resistance at the load terminals when the independent sources are deactivated. i.e. replace voltage source by short circuit & current source by an open circuit.

$$\boxed{I_L = \frac{V_{th}}{R_{th} + R_L}}$$



$V_{th} = V_{oc}$ = Thevenin's voltage or open circuit voltage

R_{th} = Thevenin's resistance or equivalent resistance

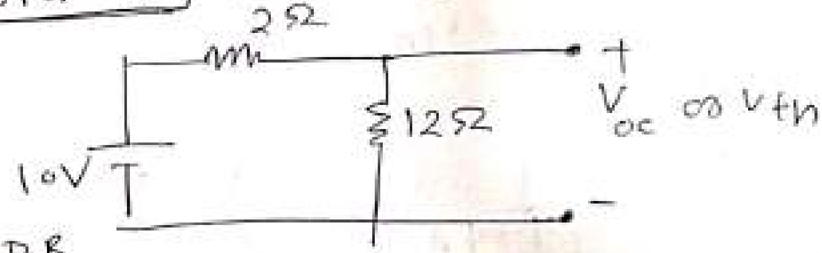
Q) 

Calculate the current in 24Ω resistor by using thevenin theorem?

A: load resistance = $R_L = 24\Omega$

i) $\boxed{\text{Step-1}}$ → Disconnect the load resistance 24Ω.

ii) $\boxed{\text{Step-2}}$ → Find $V_{oc} = V_{th}$ across open circuit terminals.

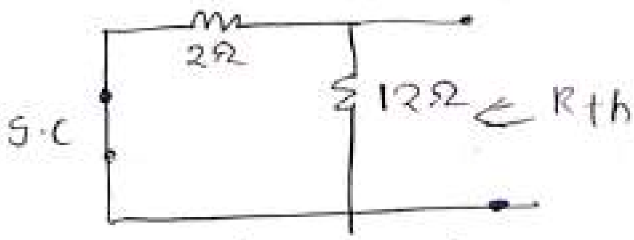


By V.D.R

$$V_{12\Omega} = V_{oc} = 10 \times \frac{12}{14} = 8.57\,V$$

iii) $\boxed{\text{Step-3}}$ → Calculate R_{th}

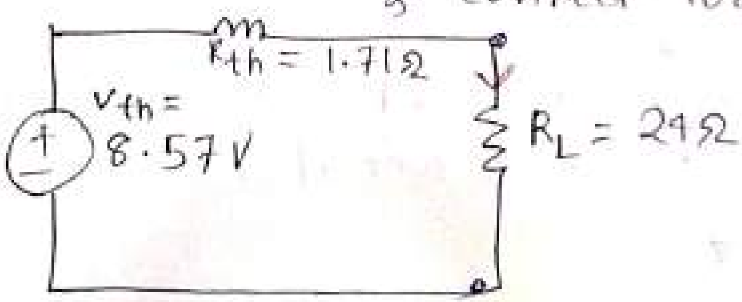
→ deactivate all the sources i.e. V.S → S.C
C.S → O.C



$$R_{th} = \frac{12 \times 2}{14} = 1.71\Omega$$

(12Ω ∥ 2Ω)

iv) $\boxed{\text{Step-4}}$ → Form thevenin's equivalent circuit & Connect load resistance in open circuit.

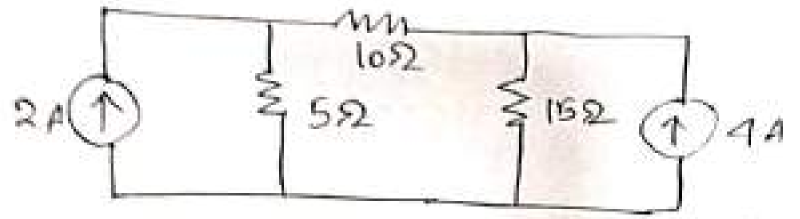


6

Step-5 Find current through 'R_L'

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{8.57}{1.71 + 24} = 0.33\,Amp.$$

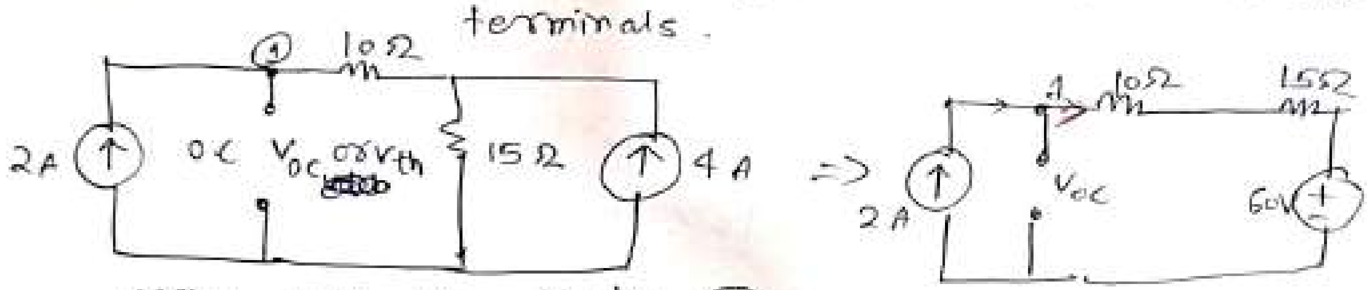
Q) Using Thevenin's theorem find the current through 5Ω resistor?



Soln - $R_L = 5\Omega$ = load resistance.

i) Step-1 → disconnect load resistance 5Ω.

ii) Step-2 → Find V_{oc} or V_{th} across open circuit terminals.

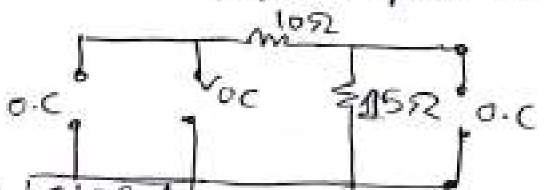


Apply KCL at Node 1

$$\frac{V_{oc} - 60}{25} = 2 \Rightarrow V_{oc} = 110\,V$$

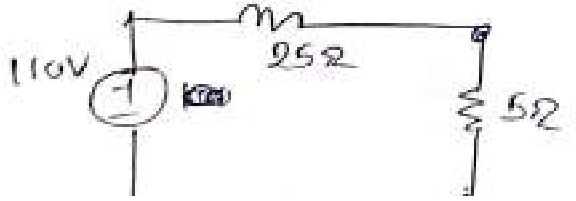
iii) Step-3 → Find R_{th}

→ All the current sources are deactivated. i.e. open circuited.



$$R_{th} = (10 + 15)\,\Omega = 25\,\Omega$$

iv) Step-4 Forming thevenin equivalent ckt & connecting load resistance in series with it.

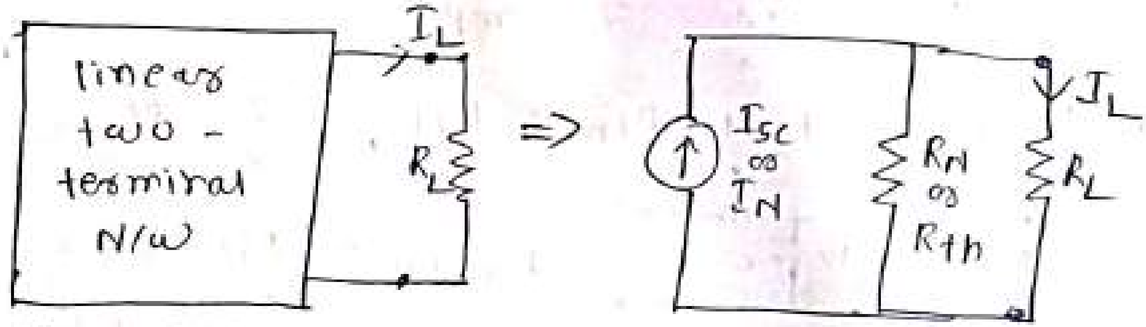


(V) Step-5 Calculate current through R_L

$$I_L = \frac{V_{th}}{R_{th}+R_L} = \frac{110}{(25+5)\Omega} = \frac{110}{30} A = 3.67 \text{ Amp (Ans)}$$

NORTON'S THEOREM

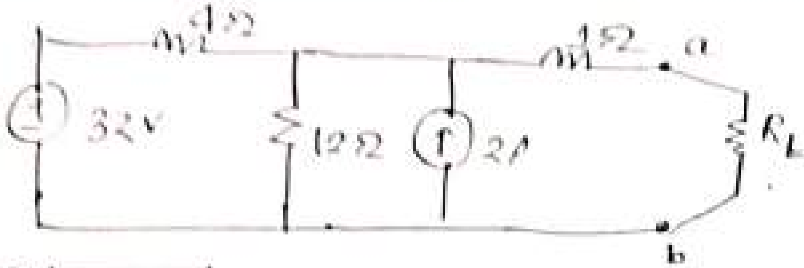
Statement: It states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_{SC} or I_N in parallel with a resistance R_N or R_{th}, where I_{SC} is the short circuit current through the load terminals ~~&~~ or R_N is the input or equivalent resistance at the load terminals when the independent sources are deactivated i.e. Voltage source short circuited & current source open circuited.



According to current divison rule

$$\boxed{I_L = I_{SC} \times \frac{R_N}{R_N + R_L}}$$

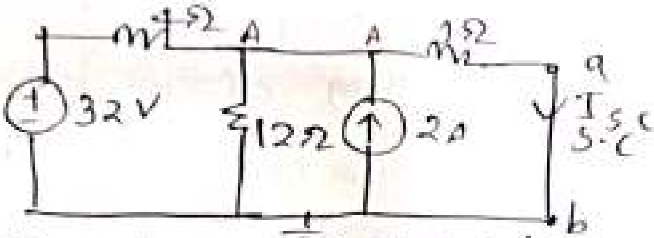
Q) (8)



Find norton current in the circuit?

A: (i) $\boxed{\text{Step-1}}$ disconnect load resistance 'R_L'

(ii) $\boxed{\text{Step-2}}$ short the two terminals 'a' & 'b' & Find short circuit current I_{sc}.



Apply KCL at Node 'A'

$$\frac{V_A - 32}{4} + \frac{V_A}{12} + \frac{V_A}{1} = 2$$

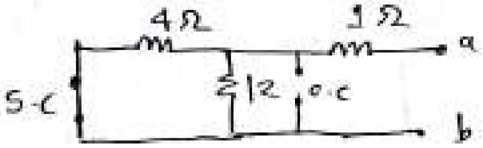
$$\Rightarrow \frac{3(V_A - 32) + V_A + 12V_A}{12} = 2$$

$$\Rightarrow 3V_A - 96 + 13V_A = 24$$

$$\Rightarrow 16V_A = 120 \quad \Rightarrow V_A = \frac{120}{16} = \frac{15}{2} = 7.5 \text{ volt}$$

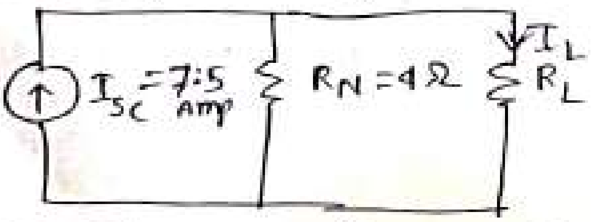
$$I_{SC} = \frac{V_A}{1} = 7.5 \text{ Amp}$$

(iii) $\boxed{\text{Step-3}}$ Find R_{th} or R_N, O.C = 2A, S.C = 32V



$$R_{th}/R_N = (4 \parallel 12) + 1$$
$$= \frac{12 \times 4}{16} + 1 = 4\Omega$$

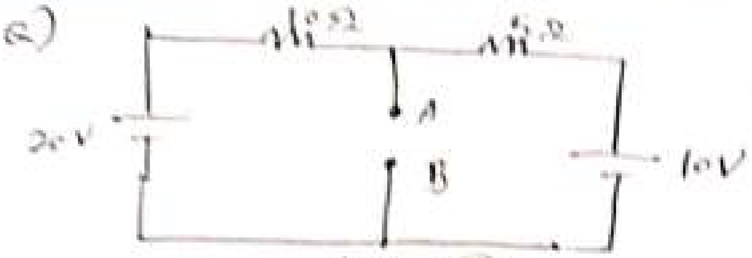
(iv) $\boxed{\text{Step-4}}$ Forming Norton equivalent circuit



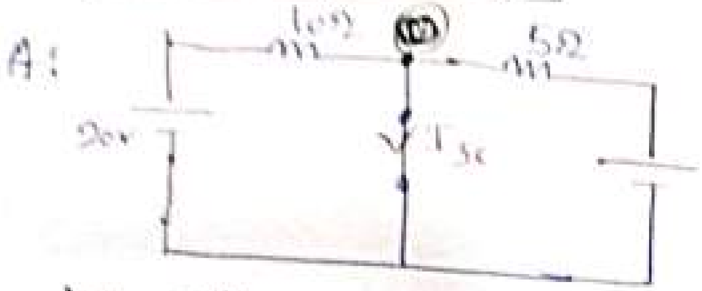
(v) $\boxed{\text{Step-5}}$ Find I_L $= I_{SC} \times \frac{4}{(R_L + 4)}$

$$= 7.5 \times \frac{4}{R_L + 4} \text{ Amp (ANS)}$$

Q) Determine Norton's equivalent ckt ?



A:



$\cancel{\frac{V-20}{10} + \frac{V-10}{5} + I_{sc} = 0}$

$\Rightarrow \cancel{\frac{V-20}{10} + \frac{2V-20}{10}} = -I_{sc}$

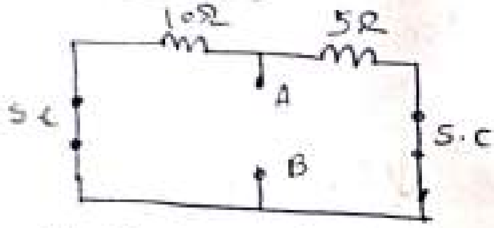
by SPT

$I_1 = \frac{20}{10} = 2\,Amp$

$I_2 = \frac{10}{5} = 2\,Amp$

$\Rightarrow I = I_1 + I_2 = 4\,Amp$

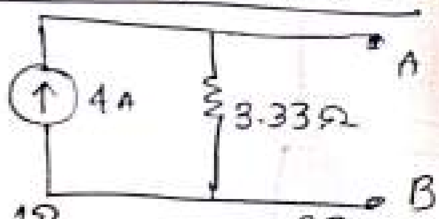
Rth / RN



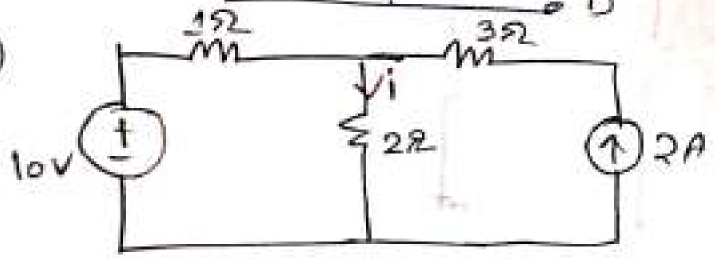
$R_{th} = (10 \parallel 5)\Omega$

$= \frac{10 \times 5}{15} = \frac{10}{3} = 3.33\,Amp$

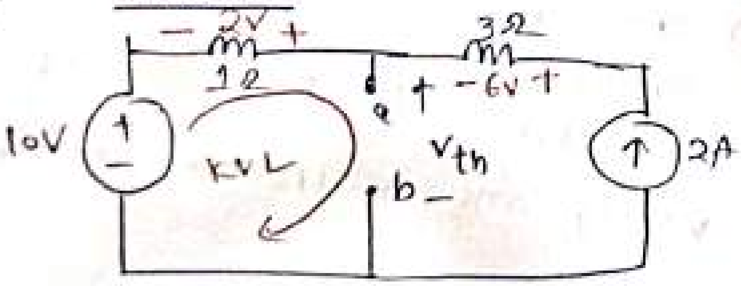
Norton equivalent ckt



Q) Find current in 2Ω resistor by using thevenin & Norton's theorem's.



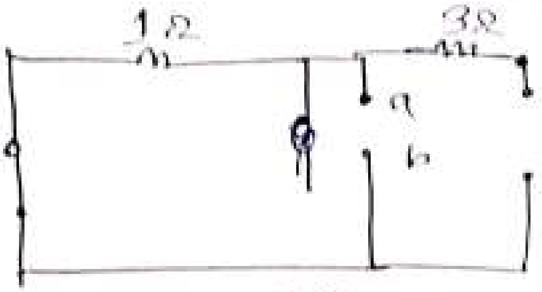
A: Thevenin



$10 + 2 - V_{th} = 0$

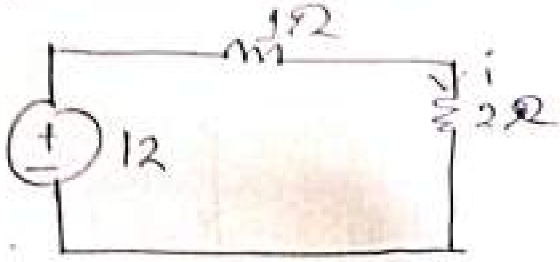
$\boxed{V_{th} = 12\,V}$

<u>R_{th}</u>



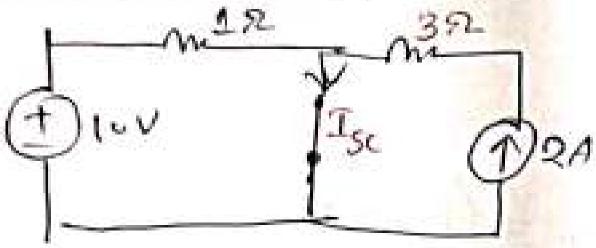
O·C $R_{th}/R_N = 1\Omega$

<u>TEN</u>



$$I = \frac{12\ V}{(2+1)\Omega} = 4\ Amp$$

<u>Norton's theorem</u>



<u>by SPT</u>

<u>2A</u>

$I_1 = I_{SC} = 2A$

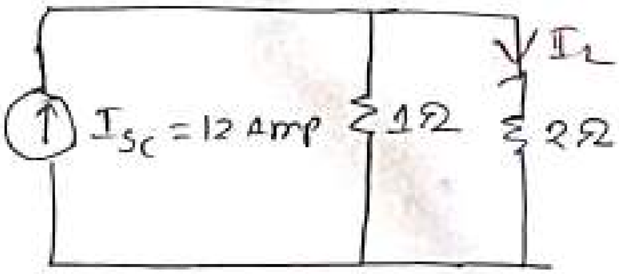
<u>10V</u>

$I_2 = I_{SC} = \frac{10}{1} = 10 Amp$

$I_{SC} = 12\ Amp$

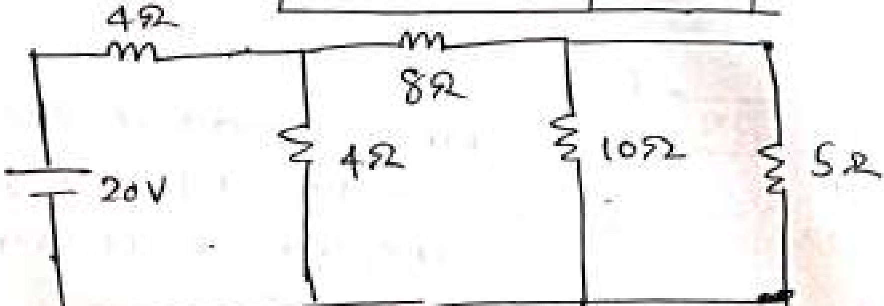
$R_N/R_{th} = 1\Omega$

<u>NEN</u>



$$I_L = 12 \times \frac{1}{3}$$
$$= 4\ Amp.$$

Q) H.W

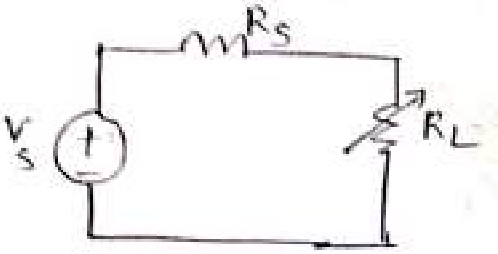


Apply i) Norton's theorem
ii) Thevenin theorem
iii) Nodal Analysis
iv) Mesh analysis
to find current in 5Ω resistor & verify answer in all the '4' methods?

A: 0.5 Amp (ANS).

Maximum power transfer theorem

Statement :- It states that maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.



Assume that the load resistance is variable.

According to MPTT, maximum power is transferred from source to a load, if $\boxed{R_S = R_L}$

R_S = source resistance

R_L = load resistance

Proof :- current in the above circuit $I = \dfrac{V_S}{(R_S + R_L)}$

power delivered to the load R_L is $P = I^2 R_L$

$$P = \frac{V_S^2 R_L}{(R_S + R_L)^2} \quad \cdot (1)$$

→ To determine the value of R_L for maximum power to be transferred to the load, we have to set the 1st derivative of eqn (1) with respect to R_L, i.e. when $\dfrac{dP}{dR_L} = 0$.

$$\Rightarrow \frac{dP}{dR_L} = \frac{d}{dR_L}\left(\frac{V_S^2 R_L}{(R_S+R_L)^2}\right) = 0 \qquad \frac{d\left(\frac{u}{v}\right)}{} = \frac{u'v - v'u}{v^2}$$

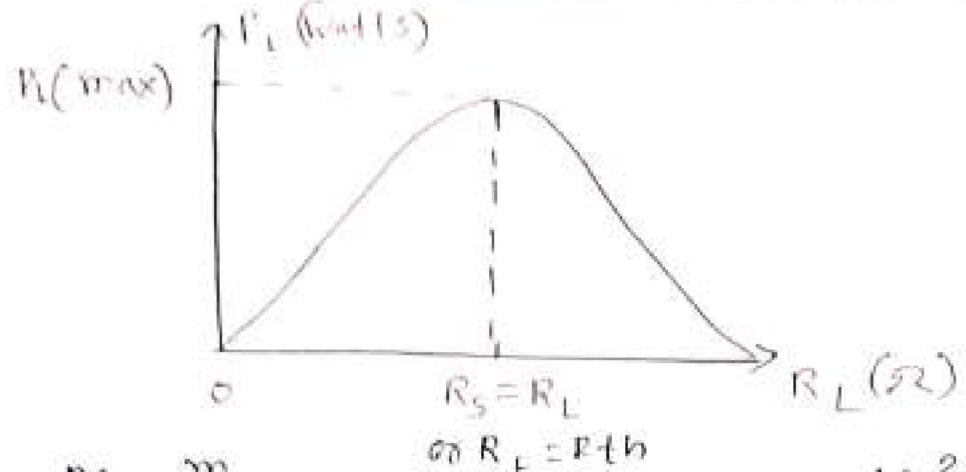
$$\Rightarrow \frac{V_S^2\{(R_S+R_L)^2 - 2(R_S+R_L)R_L\}}{(R_S+R_L)^4} = 0$$

$$\Rightarrow (R_S+R_L)^2 - 2R_L(R_S+R_L) = 0$$

$$\Rightarrow R_S^2 + R_L^2 + 2R_SR_L - 2R_SR_L - 2R_L^2 = 0$$

$$\Rightarrow R_S^2 - R_L^2 = 0 \quad \Rightarrow \boxed{R_S = R_L}$$

∴ so maximum power is transferred to the load when load resistance is equal to source resistance.



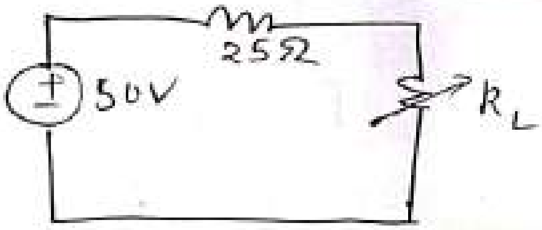
Max^m power is $P_{L(max)} = \dfrac{V_S^2 R_L}{(R_S+R_L)^2}$

Put $R_S = R_L$, $P_{L(max)} = \dfrac{V_S^2 R_L}{4 R_L^2}$

$$\boxed{\therefore P_L \max = \frac{V_S^2}{4 R_L} \text{ watt}}$$

or $\boxed{P_{max} \text{ occurs in } R_L \text{ when } R_L = R_{th} \;\&\; P_{max} = \dfrac{V_{th}^2}{4R_{th}}}$

Q)



Determine the value of load resistance when the load th draws maximum power & also find the maximum power?

A: power is maximum at the load when load resistance is equal to source resistance.

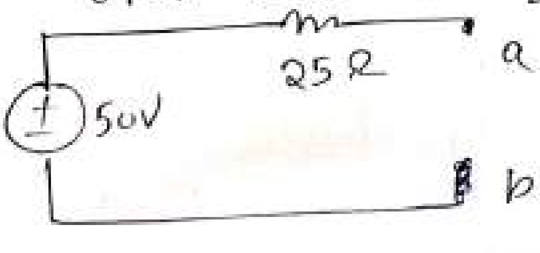
$\therefore R_L = 25\Omega$, $I = \dfrac{50}{25+25} = 1$ Amp

$P_{max} = I^2 R_L$

$= (1)^2 \times 25 = 25$ watt (ANS)

or) find R_{th} & V_{th}

open circuit P_L



$V_{th} = 50V$

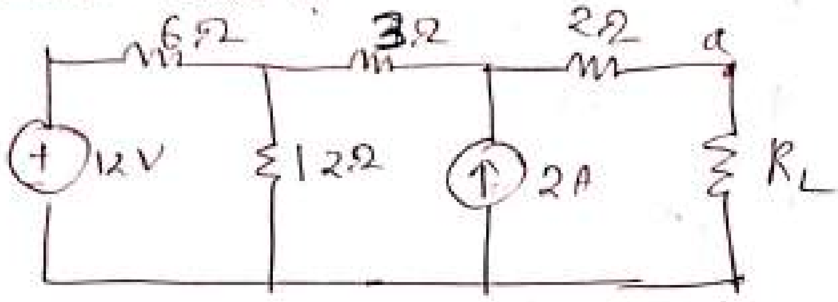
$R_{th} = 25\Omega$

$P_{max} = \dfrac{V_{th}^2}{4R_{th}}$

$= \dfrac{2500}{4\times 25}$

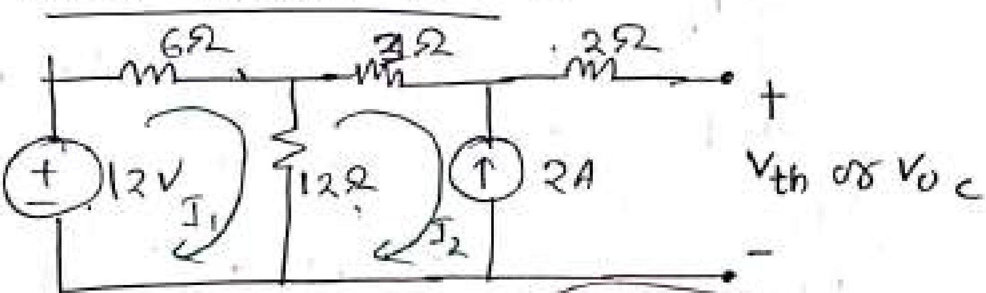
$= 25$ watt

Q) Find load resistance R_L for maximum power transfer in the given circuit. Find the maximum power also?



A: We need to find thevonin voltage V_{th} & thevonin resistance R_{th} across the terminals a-b.

open circuit R_L :



Here we can see $I_2 = -2A$

Apply KVL in mesh-①

$12 - 6I_1 - 12(I_1 - I_2) = 0$

$12 - 6I_1 - 12I_1 + 12I_2 = 0$

$\Rightarrow 12 = 18I_1 - 12(-2)$ $\Rightarrow 12 - 24 = 18I_1$

$\Rightarrow -12 = 18I_1$

$\boxed{I_1 = -\frac{12}{18} = -\frac{2}{3} \text{ Amp}}$

Apply KVL around the outer loop to get V_{th}

$12 - 6I_1 - 1I_2 - 2 \times 0 - V_{th} = 0$

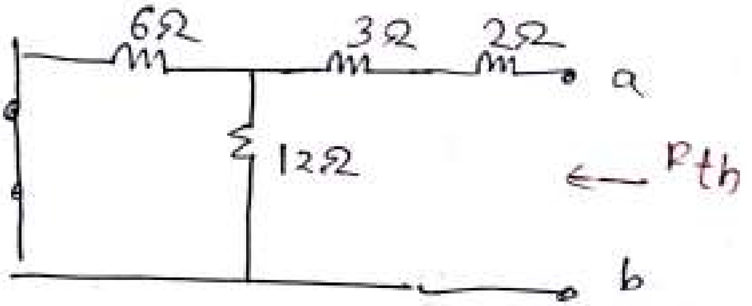
$12 - 6(-2/3) - 3(-2) = V_{th} \Rightarrow V_{th} = 12 + 4 + 6$

$= 22 V$

(14)

Find R_{th}

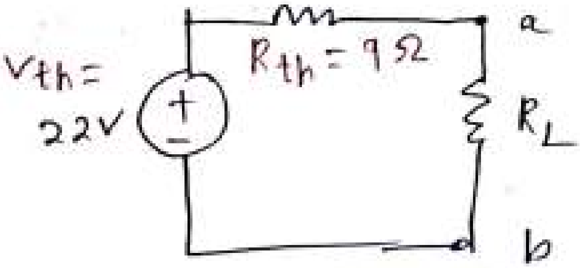
O.C = 2A
S.C = 12V



$$R_{th} = \left[(12 \parallel 6) + 3 + 2\right]\Omega$$

$$= \left[\frac{12\times 6}{18} + 5\right]\Omega = 9\Omega$$

Thevenin equivalent circuit



we know max^m power is transferred from source to load when $R_L = R_{th} = 9\Omega$.

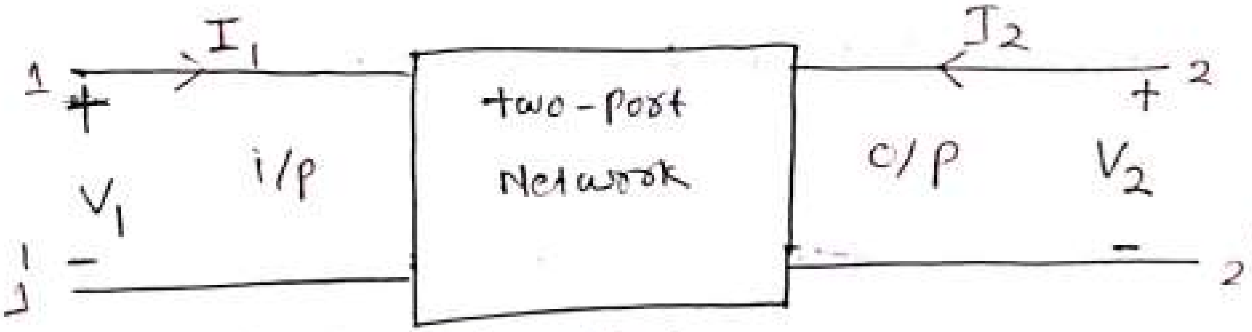
$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{(22)^2}{4\times 9} = \frac{\overset{11}{22}\times\overset{11}{22}}{\underset{2}{4}\times 9} = \frac{121}{9}$$

$= 13.44$ (watt)
(ANS)

Two - Port Network

Port : A port is defined as any pair of terminals at which a signal may enter or leave a Network. ~~is called a port~~.

→ A Network with two i/p terminals and two o/p terminals is called a four - terminal N/w or a two - port Network.



V_1 = input port voltage

V_2 = output port voltage

I_1 = input port current

I_2 = output port current

$1 - 1'$ = input port

$2 - 2'$ = output port

Relationship of two - port Variables

max^m No. of possible parameters for analysis of two port N/w is given by $^4C_2 = \frac{4 \times 3}{1 \times 2} = 6$.

Six sets of two - port parameter for a given N/w.

(1) open - circuit impedance parameters or Z - parameters.

(2) Short-circuit Admittance parameters or (Y-parameter)

(3) Transmission parameters or ABCD-parameter or T-parameter

(4) Hybrid parameters (h-parameters)

(5) Inverse transmission parameters

(6) inverse hybrid parameters.

Symmetrical Network

A two port N/w is said to be symmetrical if the ratio of excitation to response at both the ports independently are equal w.r.t the defined circuit condition such as open circuit or short. ckt. or its i/p impedance = o/p impedance.

Fig (a)

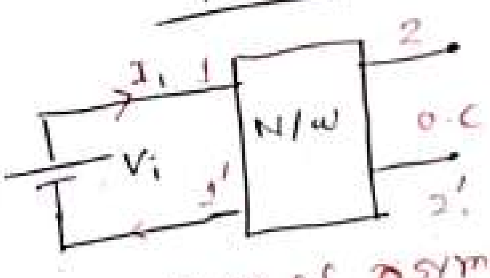
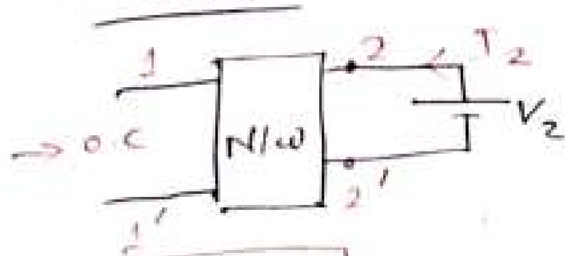


Fig (b)



condition of symmetry $\boxed{\frac{V_1}{I_1} = \frac{V_2}{I_2}}$

Concept of reciprocity

if the port-2 voltage due to applied current at port-1 is the same as the port-1 voltage when the applied current is same as port-2.

(1) Open-circuit impedance parameters (Z-parameters)

→ The impedance parameter represent the relation between the voltages and the currents in the two port Network.

→ in Z-parameter, the input voltage V_1 and current I_1 & O/P voltage V_2 and I_2 are expressed as follows.

$$\boxed{\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \cdot \text{(i)} \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \cdot \text{(ii)} \end{aligned}} \text{ or } \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

impedance parameter matrix

Now put $I_2 = 0$ (secondary side open circuited) in eqn (i) & (ii)

$V_1 = Z_{11} I_1 \Rightarrow \boxed{Z_{11} = \frac{V_1}{I_1} \Big|_{I_2 = 0}}$ = driving point impedance at port-1

$V_2 = Z_{21} I_1 \Rightarrow \boxed{Z_{21} = \frac{V_2}{I_1} \Big|_{I_2 = 0}}$ = Transfer impedance

Now put $I_1 = 0$ in eqn (i) & (ii) (primary side open circuited)

$V_1 = Z_{12} I_2 \Rightarrow \boxed{Z_{12} = \frac{V_1}{I_2} \Big|_{I_1 = 0}}$ = Transfer impedance

$V_2 = Z_{22} I_2 \Rightarrow \boxed{Z_{22} = \frac{V_2}{I_2} \Big|_{I_1 = 0}}$ = driving point impedance at port-2

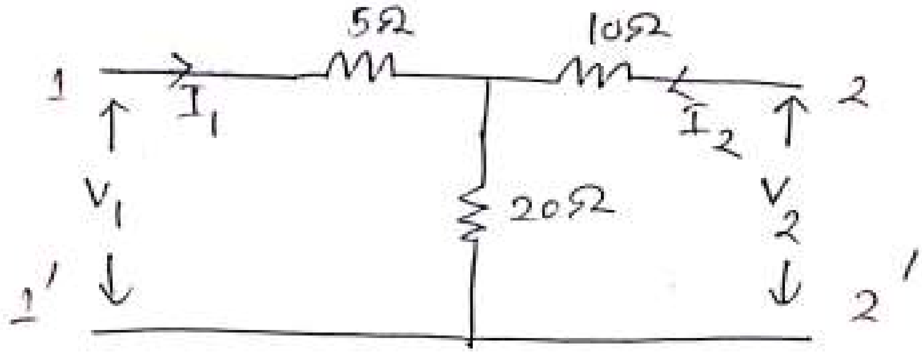
Condition for (i) Reciprocal Network

$$\boxed{Z_{12} = Z_{21}} \checkmark$$

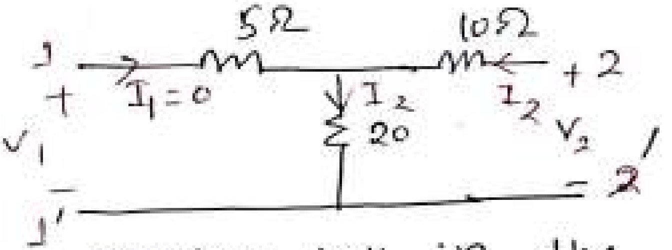
(ii) symmetry Network

$$\boxed{Z_{11} = Z_{22}} \checkmark$$

Q) Calculate the Z-Parameters of the Network ?



Ans: (a) consider $I_1 = 0$, i.e. port-1 is open circuited.



Applying KVL in the right mesh

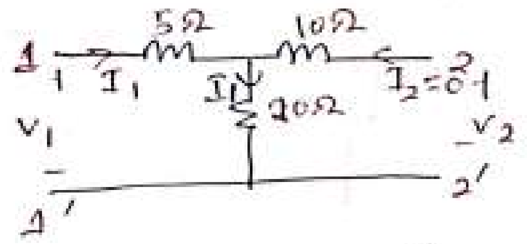
$$-10I_2 - 20I_2 + V_2 = 0$$

$$\Rightarrow V_2 = 30I_2 \Rightarrow \left.\frac{V_2}{I_2}\right|_{I_1=0} = \boxed{Z_{22} = 30\Omega}$$

Applying KVL in the left mesh

$$V_1 - 20I_2 = 0 \Rightarrow \frac{V_1}{I_2} = 20 \Rightarrow \left.\frac{V_1}{I_2}\right|_{I_1=0} = \boxed{Z_{12} = 20\Omega}$$

(b) consider $I_2 = 0$, port-2 is open circuited.



Applying KVL in the left mesh

$$V_1 - 5I_1 - 20I_1 = 0$$

$$V_1 = 25I_1$$

$$\Rightarrow \left.\frac{V_1}{I_1}\right|_{I_2=0} = \boxed{Z_{11} = 25\Omega}$$

Applying KVL in the right mesh

$$V_2 - 20I_1 = 0 \Rightarrow \left.\frac{V_2}{I_1}\right|_{I_2=0} = \boxed{Z_{21} = 20\Omega}$$

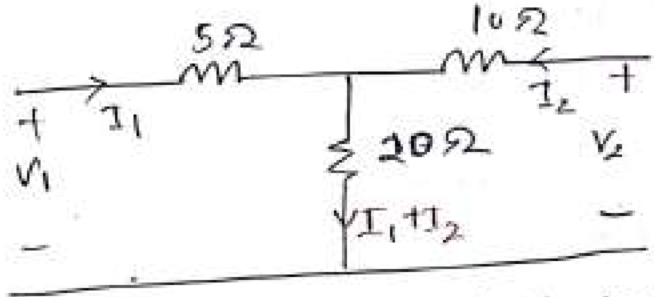
$\therefore\ V_1 = 25I_1 + 20I_2$ $\quad [Z] = \begin{bmatrix} 25 & 20 \\ 20 & 30 \end{bmatrix} \Omega$

$V_2 = 20I_1 + 30I_2$

$Z_{12} = Z_{21}$ = reciprocal N/W

$Z_{11} \neq Z_{12}$ = Not symmetrical N/W

or)



we want to derive V_1 & V_2 in terms of I_1 & I_2.

A $V_1 - 5I_1 - 20(I_1+I_2) = 0$ (KVL in left mesh)

$V_1 - 5I_1 - 20I_1 - 20I_2 = 0$

$\boxed{V_1 = 25I_1 + 20I_2}$ ·(i) $\quad V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 - 10I_2 - 20(I_1+I_2) = 0$ (KVL in right mesh)

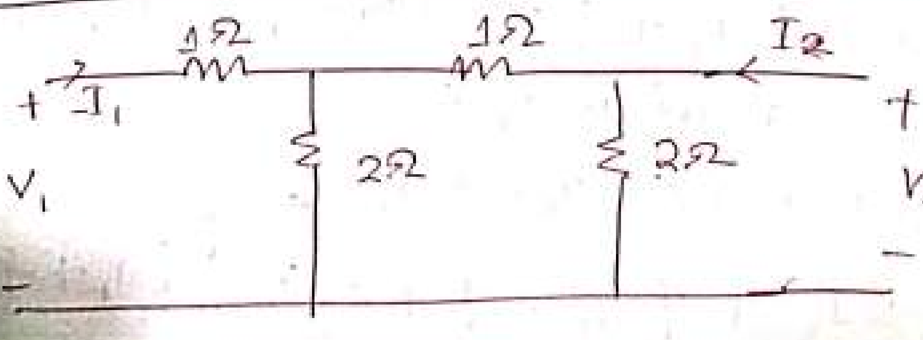
$\boxed{V_2 = 20I_1 + 30I_2}$ ··(ii) $\quad V_2 = Z_{21}I_1 + Z_{22}I_2$

comparing eq^n (i) & (ii) with Normal eq^n

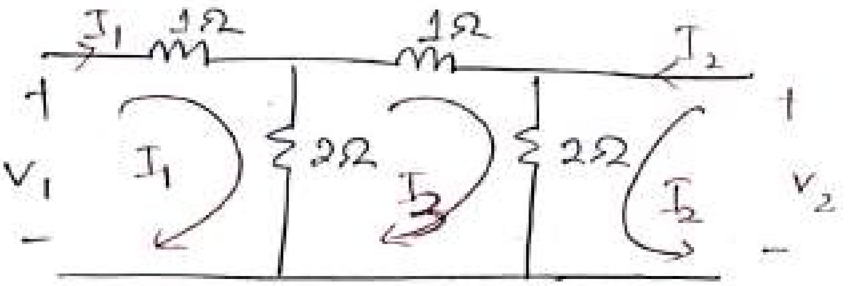
$Z_{11} = 25I_1$, $Z_{12} = 20\Omega$, $Z_{21} = 20\Omega$, $Z_{22} = 30\Omega$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 25 & 20 \\ 20 & 30 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Q) Find 'z' parameters of the following circuit



A :



Apply KVL in loop 1

$$V_1 - I_1 - 2(I_1 - I_3) = 0$$
$$V_1 - I_1 - 2I_1 + 2I_3 = 0$$
$$\Rightarrow V_1 = 3I_1 - 2I_3 \quad \ldots (i)$$

Apply KVL in loop '3'

$$-2(I_3 - I_1) - I_3 - 2(I_3 + I_2) = 0$$
$$-2I_3 + 2I_1 - I_3 - 2I_3 - 2I_2 = 0$$
$$2I_1 - 2I_2 - 5I_3 = 0 \quad \ldots (iii)$$

Apply KVL in loop '2'

$$-2(I_2 + I_3) + V_2 = 0 \Rightarrow -2I_2 - 2I_3 + V_2 = 0$$
$$\Rightarrow V_2 = 2I_2 + 2I_3 \quad \ldots (ii)$$

From eqn (iii)

$$5I_3 = 2I_1 - 2I_2$$
$$I_3 = \frac{2}{5}I_1 - \frac{2}{5}I_2 \quad \ldots (iv)$$

substitute eqn (iv) in eqn (i)

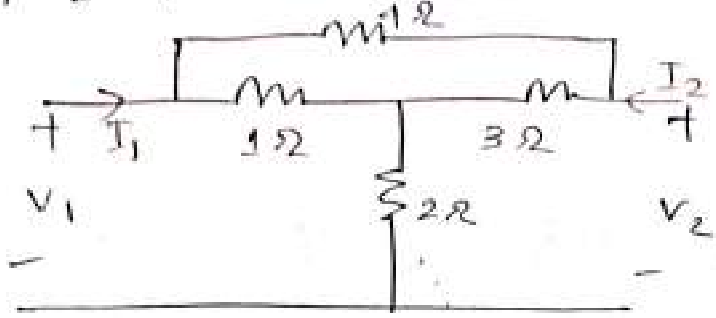
$$V_1 = 3I_1 - 2\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right)$$
$$= 3I_1 - \frac{4}{5}I_1 + \frac{4}{5}I_2 \qquad \boxed{V_1 = \frac{11}{5}I_1 + \frac{4}{5}I_2}$$

substitute eqn (iv) in eqn (ii)

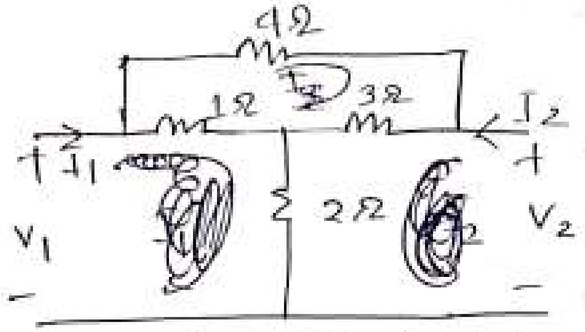
$$V_2 = 2I_2 + 2\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right)$$
$$V_2 = 2I_2 + \frac{4}{5}I_1 - \frac{4}{5}I_2 \Rightarrow \boxed{V_2 = \frac{4}{5}I_1 + \frac{6}{5}I_2}$$

$$\begin{bmatrix} Z_{11} & Z_{21} \\ Z_{12} & Z_{22} \end{bmatrix} = \begin{bmatrix} 11/5 & 1/5 \\ 1/5 & 6/15 \end{bmatrix} \Omega$$

a) Find 'Z' Parameter for the Network.



A:



Apply KVL in loop 1

$$V_1 - (I_1 - I_3) - 2(I_1 + I_2) = 0$$

$$V_1 - I_1 + I_3 - 2I_1 - 2I_2 = 0$$

$$V_1 = 3I_1 + 2I_2 - I_3 \quad \ldots (i)$$

Applying KVL in loop 2

$$V_2 - 3(I_2 + I_3) - 2(I_2 + I_1) = 0$$

$$V_2 - 3I_2 - 3I_3 - 2I_2 - 2I_1 = 0$$

$$\Rightarrow V_2 = 2I_1 + 5I_2 + 3I_3 \quad \ldots (ii)$$

Applying KVL in loop 3

$$-4I_3 - 3(I_2 + I_3) - (I_3 - I_1) = 0$$

$$-4I_3 - 3I_2 - 3I_3 - I_3 + I_1 = 0$$

$$\Rightarrow I_1 - 3I_2 - 8I_3 = 0$$

Put eq (iii) in eq (i) & (ii) $\Rightarrow$ $I_3 = \frac{I_1}{8} - \frac{3}{8} I_2 \quad \ldots (iii)$

$$V_1 = \frac{23}{8} I_1 + \frac{19}{8} I_2$$

$$V_2 = \frac{19}{8} I_1 + \frac{31}{8} I_2$$

Short circuit admittance Parameters or (Y- Parameters)

→ The admittance parameters represent the relation between the currents & voltages in the two-port Network.

→ The admittance parameter matrix may be written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}}_{\text{Admittance matrix}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{or} \quad \begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned}$$

Now take $V_2 = 0$ (secondary side short circuited)

$I_1 = Y_{11}V_1 \Rightarrow$

$\Rightarrow \boxed{Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}}$ = driving point Admittance at port-1

$I_2 = Y_{21}V_1$

$\Rightarrow \boxed{Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}}$ = Transfer admittance.

take $V_1 = 0$ (primary side short circuited)

$I_1 = Y_{12}V_2 \Rightarrow \boxed{Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}}$ = transfer admittance.

$I_2 = Y_{22}V_2 \Rightarrow \boxed{Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}}$ = driving point Admittance at port-2

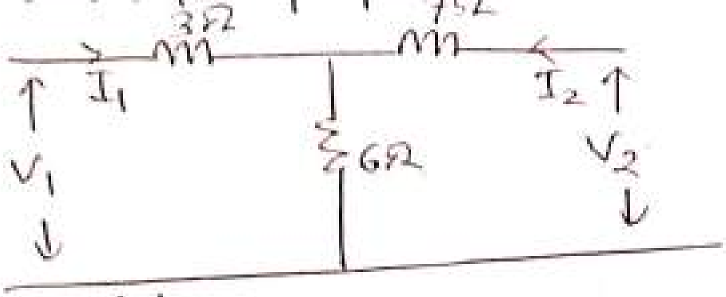
condn for reciprocity

$$Y_{12} = Y_{21}$$

condn for symmetry

$$Y_{11} = Y_{22}$$

6) calculate Y-parameter



A: Step-1
Calculation of Y_{11} & Y_{21} : Put $V_2 = 0$ and its circuit secondary side short circuited and its circuit is given

$$V_1 = [(6 \| 7)\Omega + 3]\Omega\, I_1$$
$$= \left(\frac{6 \times 7}{13} + 3\right)\Omega\, I_1$$
$$= \left(\frac{42}{13} + 3\right) I_1$$
$$= \left(\frac{42+39}{13}\right) I_1 = \frac{81}{13} I_1$$

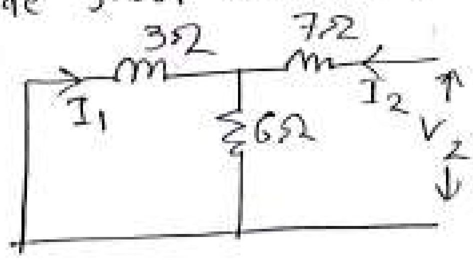
$$Y_{11} = \frac{13}{81} \text{ mho}$$

by using current divison rule, $I_2 = -I_1 \times \left(\frac{6}{6+7}\right)$

$$\boxed{I_2 = -\frac{6}{13} I_1}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-I_1\left(\frac{6}{6+7}\right)}{\frac{81}{13} I_1} = \frac{-6}{81} \text{ mho}$$

Step-II calculation of Y_{12} & Y_{22}. Put $V_1 = 0$ (primary side short circuted.



$$V_2 = I_2\left[(3 \| 6) + 7\right] = 9 I_2$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$
$$= \frac{I_2}{9 I_2} = \frac{1}{9} \text{ mho}$$

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} = \frac{-2/3\, I_2}{9 I_2} = -\frac{2}{27}\text{ mho}$$

since $I_1 = -I_2 \times \left(\frac{6}{6+3}\right) = \frac{-2}{3} I_2$

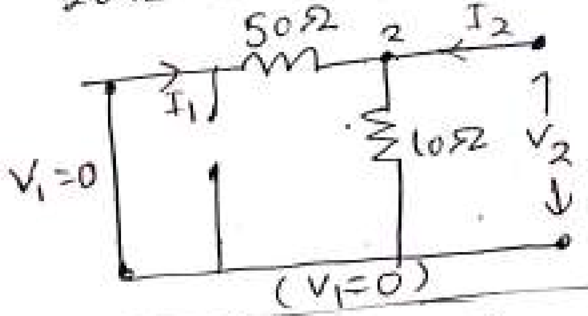
$Y_{11} = \frac{13}{81}\,\mho$, $Y_{12} = -\frac{2}{3}\,\mho$, $Y_{21} = \frac{-6}{81}\,\mho$, $Y_{22} = \frac{1}{9}\,\mho$

$$I_1 = \frac{13}{81} V_1 + \left(\frac{-2}{3}\right) V_2$$

$$I_2 = \left(\frac{-6}{81}\right) V_1 + \frac{1}{9} V_2$$

Q) Find Y-parameter for the Network shown in fig 1.2.

A: when $V_1 = 0$, i.e. port-1 is short circuited. In this case, no current will flow through the 20Ω resistor.



by KCL at Node '2'

$$\frac{V_2}{10} + \frac{V_2}{50} = I_2$$

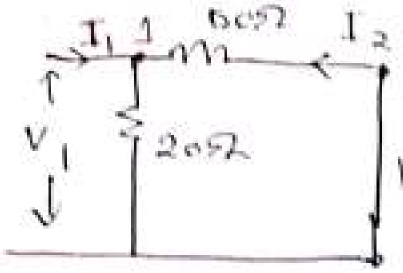
$$\Rightarrow \frac{5V_2 + V_2}{50} = I_2$$

$$\Rightarrow \frac{6V_2}{50} = I_2$$

$$\boxed{Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} = \frac{6}{50} = 0.12\,\mho}$$

$$I_1 = \frac{0 - V_2}{50} = \frac{-V_2}{50} \Rightarrow \frac{I_1}{V_2} = Y_{12} = \frac{-1}{50} = -0.02\,\Omega$$

current gain



$\boxed{V_2=0}$ i.e. port 2 is short circuited.

by KCL at Node-1

$$\frac{V_1}{20}+\frac{V_1}{50}=I_1$$

$$Y_{11}=\left|\frac{I_1}{V_1}\right|_{V_2=0}=\frac{1}{20}+\frac{1}{50}=0.07\,\Omega$$

$$I_2=\frac{0-V_1}{50} \quad \therefore Y_{21}=\left|\frac{I_2}{V_1}\right|_{V_2=0}=\frac{1}{50}=0.02\,\Omega$$

Therefore, the Y- parameters of the N/w are

$$[Y]=\begin{bmatrix}0.07 & 0.02\\ 0.02 & 0.12\end{bmatrix}\mho$$

(3) Transmission parameters (ABCD - Parameters)

→ ABCD parameters represent the relation b/w the i/p quantities & the o/p quantities in the two-port Network.

$$\begin{bmatrix}V_1\\ I_1\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}V_2\\ -I_2\end{bmatrix} \text{ or } \begin{aligned}V_1&=AV_2-BI_2\\ I_1&=CV_2-DI_2\end{aligned}$$

Calculation of A & C, put $I_2=0$ (secondary side open circuited)

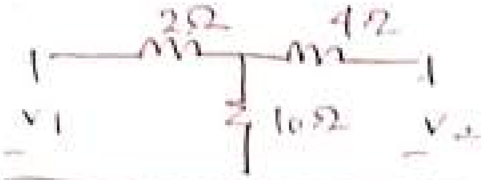
$V_1=AV_2 \Rightarrow A=\left.\frac{V_1}{V_2}\right|_{I_2=0}$ = open circuit reverse voltage gain

$I_1=CV_2 \Rightarrow C=\left.\frac{I_1}{V_2}\right|_{I_2=0}$ = ~~short~~ open-circuit transfer impedance.

Calculation of B & D, put $V_2=0$ (secondary short circuited)

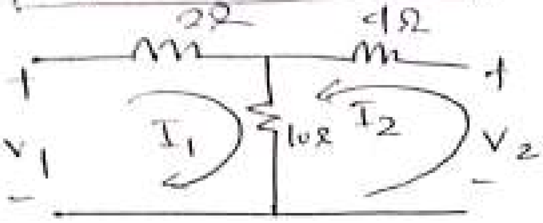
$V_1=-BI_2 \Rightarrow B=\left.\frac{V_1}{I_2}\right|_{V_2=0}$ = short-circuit transfer impedance.

$I_1=-DI_2 \Rightarrow D=\left.\frac{I_1}{I_2}\right|_{V_2=0}$ = short-circuit reverse current gain

a) 

Find ABCD Parameters ? (12)

A: $\boxed{\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}}$



Apply KVL in loop-(1)

$V_1 - 2I_1 - 10(I_1 + I_2) = 0$

$V_1 - 2I_1 - 10I_1 - 10I_2 = 0$

$\boxed{V_1 = 12I_1 + 10I_2}$. (i)

KVL in loop-(2)

$V_2 - 4I_2 - 10(I_2 + I_1) = 0$

$V_2 - 4I_2 - 10I_2 - 10I_1 = 0$

$\boxed{V_2 = 10I_1 + 14I_2}$ .. (ii)

$10I_1 = V_2 - 14I_2$

$I_1 = \frac{V_2}{10} - \frac{14}{10}I_2$

$\boxed{I_1 = 0.1V_2 - 1.4I_2}$ .. (iii)

$V_1 = 12I_1 + 10I_2$

$= 12(0.1V_2 - 1.4I_2) + 10I_2$

$= 1.2V_2 - 16.8I_2 + 10I_2$

$\boxed{V_1 = 1.2V_2 - 6.8I_2}$. (iv)

$A = 1.2$ no unit

$B = 6.8\ \Omega$

$C = 0.1\ \mho$

$D = 1.4$ no unit

Condn for reciprocity
$AD - BC = 1$
Condn for symmetry $A = D$

(b)

Parameters	Express	in terms of.
1. open-circuit impedance (Z-parameter)	V_1, V_2	I_1, I_2
2. short-circuit parameters,	I_1, I_2	V_1, V_2
3. ABCD	V_1, I_1	V_2, I_2
4. Hybrid (h)	V_1, I_2	I_1, V_2

Hybrid Parameter (h-parameter)

- The hybrid parameters represent a mixed or hybrid relation b/w the voltage & the current in the two-port N/w.

→ The h-parameter matrix may be written as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad \text{or} \quad \begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}$$

When secondary is short-circuited, $V_2 = 0$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \text{short-circuit i/p impedance}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \text{short-circuit current gain}$$

When primary is open-circuited, $I_1 = 0$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \text{open-circuit reverse voltage gain}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \text{open-circuit o/p admittance.}$$

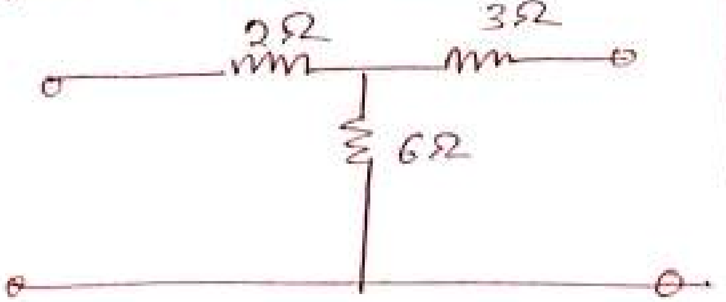
Cond^n for reciprocity

$$h_{12} = -h_{21}$$

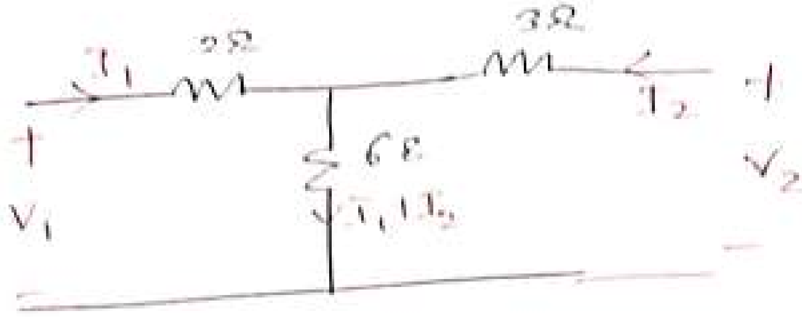
Cond^n for symmetry

$$h_{11} h_{22} - h_{12} h_{21} = 0$$

Q) Find h-parameters ?



d:



KVL in loop 2

$V_2 - 3I_2 - 6(I_1+I_2)$

$V_2 = 3I_2 + 6I_1 + 6I_2$

$V_2 = 6I_1 + 9I_2 \Rightarrow I_2 = -\frac{2}{3}I_1 + \frac{1}{9}V_2$

$h_{21} = -\frac{2}{3}, h_{22} = \frac{1}{9}\mho$

KVL in 1st loop

$V_1 - 2I_1 - 6(I_1+I_2) = 0$

$V_1 = 8I_1 + 6I_2$

$V_1 = 8I_1 + 6\left(-\frac{2}{3}I_1 + \frac{1}{9}V_2\right)$

$= 8I_1 - 4I_1 + \frac{2}{3}V_2$

$V_1 = 4I_1 + \frac{2}{3}V_2$

$h_{11} = 4\Omega \quad h_{12} = \frac{2}{3}$

$[H] = \begin{bmatrix} 4\Omega & 2/3 \\ -2/3 & \frac{1}{9}\mho \end{bmatrix}$

Inter-relation b/w Parameters (15)

Q) Express z in terms of all Parameter

a) in terms of Y

$$[Z] = \frac{1}{[Y]} = [Y]^{-1} = \frac{Adj\,[Y]}{\Delta Y}$$

$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}, \quad Adj\,[Y] = \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & Y_{11} \end{bmatrix}$$

b) in terms of [ABCD]

$$[Z] = \begin{bmatrix} \frac{A}{C} & \frac{AD-BC}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$$

c) in terms of 'h' parameter

$$[Z] = \begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$ where $\Delta h = h_{11}h_{22} - h_{12}h_{21}$

(*) Y parameters in terms of other Parameters

a) Y in terms of 'Z'

$$[Y] = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

Where

$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$

b) Y parameter in terms of Transmission (ABCD) parameter

$$[Y] = \begin{bmatrix} \frac{D}{B} & \frac{BC - AD}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$$

Y - parameter in terms of hybrid (h) parameter

$$[Y] = \begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix} \quad \Delta h = h_{11}h_{22} - h_{12}h_{21}$$

Q) Transformation of parameters

Given $[Z] = \begin{bmatrix} 10 & 20 \\ 20 & 60 \end{bmatrix}$

determine Y, ABCD & h-parameters ?

A: i) Y - parameter in terms of 'Z' parameter

$$[Y] = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix} \quad \Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$$

$$= 10 \times 60 - 20 \times 20$$

$$= 200$$

$$[Y] = \begin{bmatrix} 10/200 & -20/200 \\ \frac{-20}{200} & 60/200 \end{bmatrix} = \begin{bmatrix} 1/20 & -1/10 \\ -1/10 & 3/10 \end{bmatrix}$$

ii) Transmission parameter in terms of 'Z' parameter

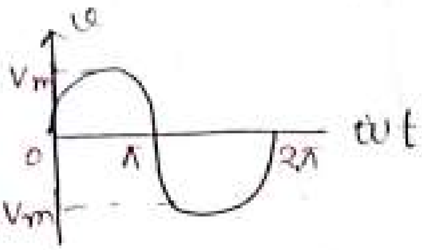
$$[T] = \begin{bmatrix} Z_{11}/Z_{21} & \Delta Z/Z_{21} \\ 1/Z_{21} & Z_{22}/Z_{21} \end{bmatrix} = \begin{bmatrix} 10/100 & 200/20 \\ 1/20 & 60/20 \end{bmatrix} = \begin{bmatrix} 1/2 & 10 \\ 1/20 & 3 \end{bmatrix}$$

iii) h - parameter in terms of 'Z'

$$[h] = \begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & 1/Z_{22} \end{bmatrix} = \begin{bmatrix} 200/60 & 20/60 \\ -20/60 & 1/60 \end{bmatrix} = \begin{bmatrix} 10/3 & 1/3 \\ -1/3 & 1/60 \end{bmatrix} \text{(Ans)}$$

AC circuit and Resonance

a) Radian



$v = V_m \sin \omega t$

V_m = Amplitude or maximum value of voltage

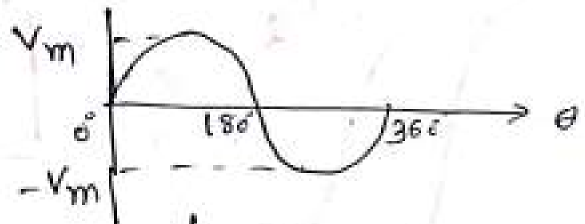
ω = Angular frequency in rad/sec

$\omega = 2\pi f$

$f = \frac{1}{T}$ $\quad \boxed{\omega = \frac{2\pi}{T}}$

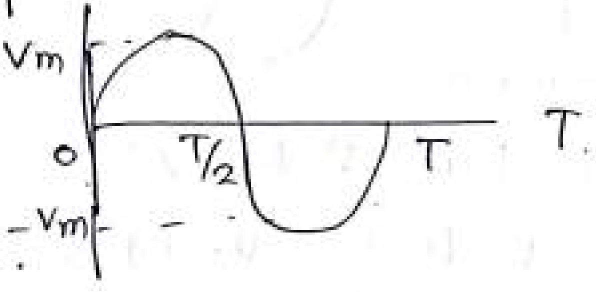
b) degree

$v = V_m \sin \theta$



c) Time

$v = V_m \sin\left(\frac{2\pi t}{T}\right)$



In India $\quad f = 50 Hz$

$T = \frac{1}{50} = 20 \text{ msec}$ $\quad (\because \pi = 3.141)$

$\omega = 2\pi f = 2 \times 50 \times \pi = 100\pi$

$\simeq 314$ rad/sec

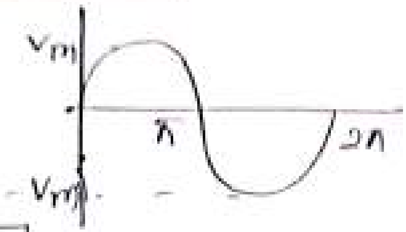
$1T = 2\pi = 360° = 20$ msec

↑ radian $\quad$ ↑ degree $\quad$ ↑ time.

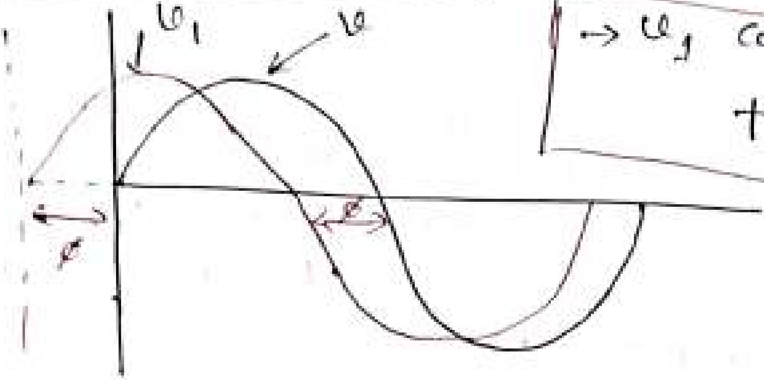
Standard sinusoidal waveform

$v = V_m \sin\omega t$

where v = instantaneous value

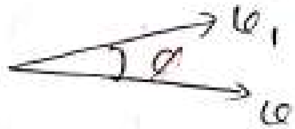


i) $\boxed{v_1 = V_m \sin(\omega t + \phi)}$



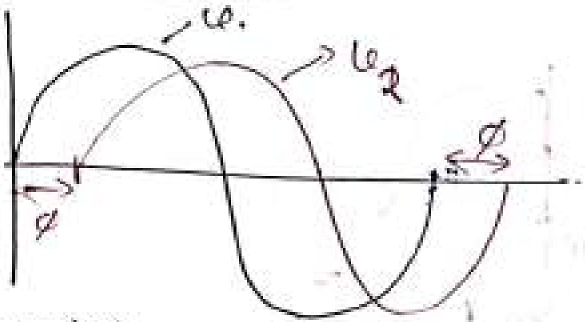
→ v_1 comes 1st then v.
$+\phi$ → leading

Phasor diagram



⇒ Conclusion : v_1 leads v by ϕ.
or v lags v_1 by ϕ.

ii) $\boxed{v_2 = V_m \sin(\omega t - \phi)}$



v_2 comes after v.
$-\phi$ = lagging

Phasor diagram

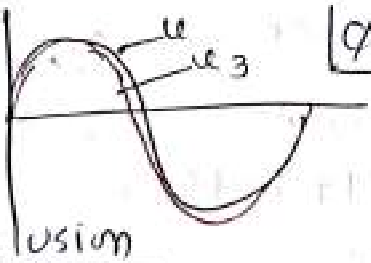
⇒ Conclusion

v_2 lags v by $\phi°$.

or v leads v_2 by ϕ.

iii) $v_3 = V_m \sin\omega t$

$\boxed{\phi = 0°}$



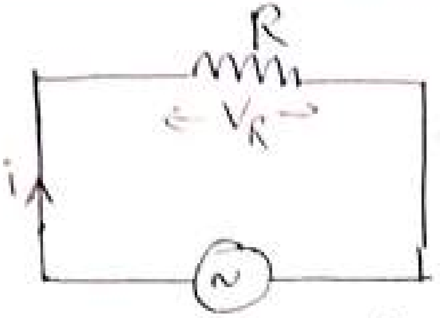
v & v_3 comes at the same time.
$\phi = 0$

Phasor diagram

Conclusion

v_3 is in phase with v.

A.C. through pure Resistance



$v = V_m \sin \omega t$

i) Let the alternating voltage applied across the circuit be given by the equation

$= v = V_m \sin \omega t$ .. (1)

where V_m = max^m value of applied voltage

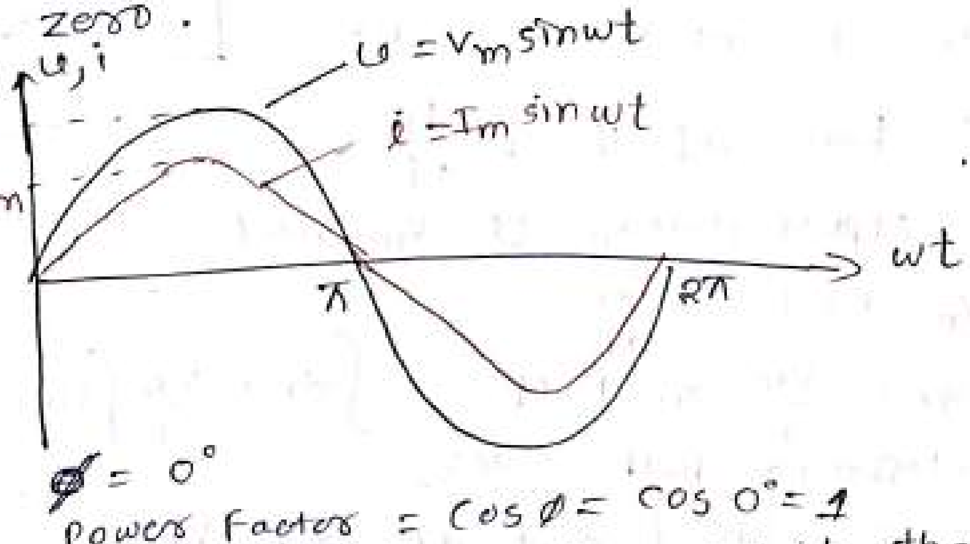
ii) Then the instantaneous value of current flowing through the resistor shown in the fig

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \left(\frac{V_m}{R}\right) \sin \omega t$$

i.e. $i = I_m \sin \omega t$ .. (ii) where $I_m = \frac{V_m}{R}$ = max^m value of current.

(iii) The value of current will be maximum when $\omega t = 90°$, $\sin \omega t = 1$

comparing eq^n (i) & (ii), we find that alternating voltage and current are in phase with each other. i.e. Phase angle b/w voltage & current is zero.



$\phi = 0°$

power factor $= \cos \phi = \cos 0° = 1$

→ Hence in pure resistive circuit, the current is in phase with the Voltage.

Power in pure resistive ckt

instantaneous power, $P = vi$

$$= V_m \sin\omega t \; I_m \sin\omega t$$

$$P = \frac{V_m I_m}{2} \, 2\sin^2\omega t$$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \left(1 - \cos 2\omega t\right)$$

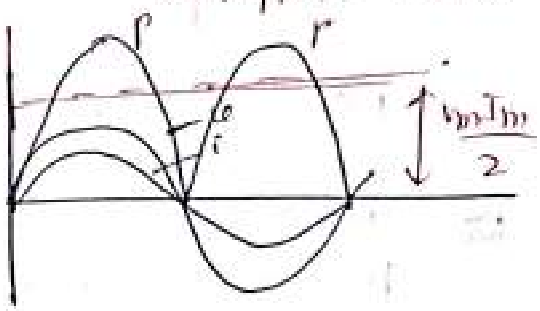
$$= \frac{V_m I_m}{\sqrt{2}\sqrt{2}} - \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \cos 2\omega t$$

This part becomes zero over a complete cycle

The avg. value of $\cos\omega t$ is zero.

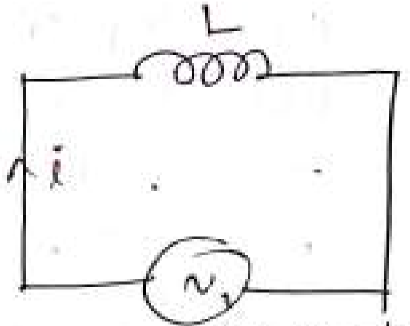
so $P_{avg} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$

$$\boxed{P_{avg} = V_{rms} \cdot I_{rms}} \text{ watt}$$



AC through pure inductance

whenever an alternating voltage is applied to a purely inductive coil, a back e.m.f is produced due to self inductance of the coil.



$v = V_m \sin\omega t$

back emf $v = L\frac{di}{dt}$

Applied voltage $v = V_m \sin\omega t$

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L} \sin\omega t \, dt \qquad \int di = \frac{V_m}{L}\int \sin\omega t \, dt$$

integrating both sides,

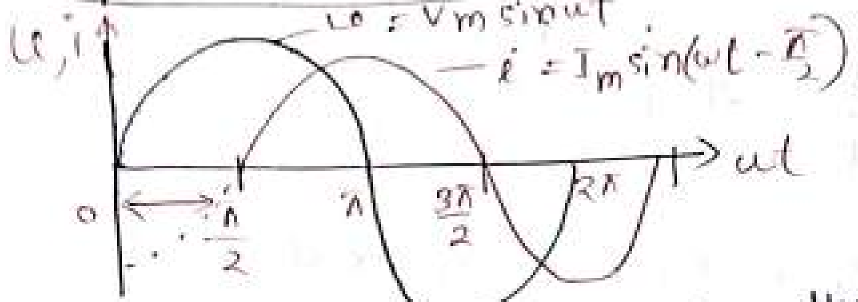
$$i = \frac{V_m}{L}\int \sin\omega t \, dt \Rightarrow i = \frac{V_m}{\omega L}(-\cos\omega t)$$

$$= -\frac{V_m}{\omega L}\cos\omega t$$

$$= \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) = \frac{V_m}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

where $I_m = \frac{V_m}{X_L}$ = max^m value of current when $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$.

$$\boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)}$$



Phase angle
$\phi = 90^\circ$
or $\pi/2$

In pure inductive circuit the current lags behind the voltage by 90°.

$$\boxed{\begin{array}{l} X_L = \omega L = 2\pi f L \text{ ohms} \\ = \text{inductive reactance} \\ \text{unit of } L = \text{Henry} \\ \omega = \text{rad/sec} \quad X_L = \text{ohm} \end{array}}$$

$$\boxed{\therefore X_L \propto f}$$

power Factor = $\cos\phi = \cos 90^\circ = 0^\circ$

power in pure 'L'

→ The avg. power consumed in a purely inductive circuit is zero.

$p = vi$

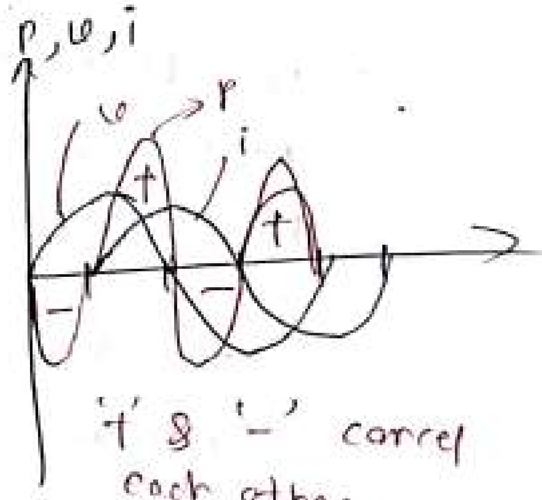
$= (V_m \sin\omega t) I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

$= V_m I_m \sin\omega t \cos\omega t$

$= \frac{V_m I_m}{2} 2 \sin\omega t \cdot \cos\omega t$

$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \sin 2\omega t$

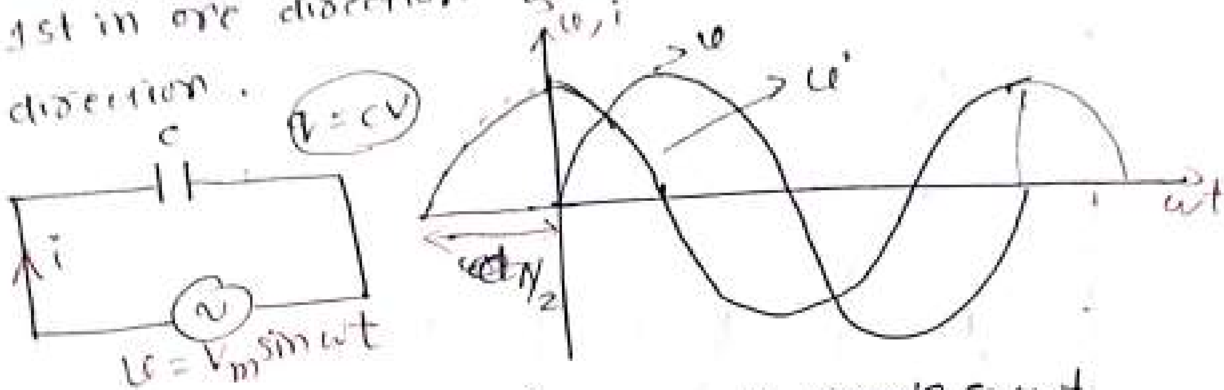
$\boxed{P = 0}$



'+' & '−' cancel each others.

AC through pure capacitance alone

when an alternating voltage is applied to the plates of a capacitor, the capacitor is charged in 1st in one direction & then in the opposite direction.



$$i = \frac{dq}{dt} = \frac{d}{dt}(C v_m \sin\omega t) = \omega C V_m \cos\omega t$$

$$= \frac{V_m}{1/\omega C} \cos\omega t$$

$$\boxed{i = \frac{V_m}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right)}$$

where $I_m = \frac{V_m}{1/\omega C}$

$= \frac{V_m}{X_C}$

$X_C = \frac{1}{\omega C}$ = capacitive reactance

unit C = Farad

X_C = ohm

ω = rad/sec

∴ In pure capacitive circuit, the current i leads voltage v by 90°.

Phase angle = $\phi = 90°$, power factor = $\cos\phi = \cos 90° = 0$

→ The average power consumed in pure capacitive circuit is zero.

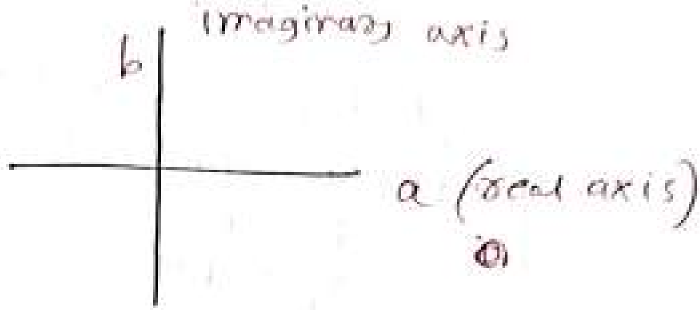
J-operator

→ j is used to indicate the counterclockwise rotation of a vector through 90°.

→ $j = \sqrt{-1}$, $j^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$, $j^3 = -j$, $j^4 = +1$

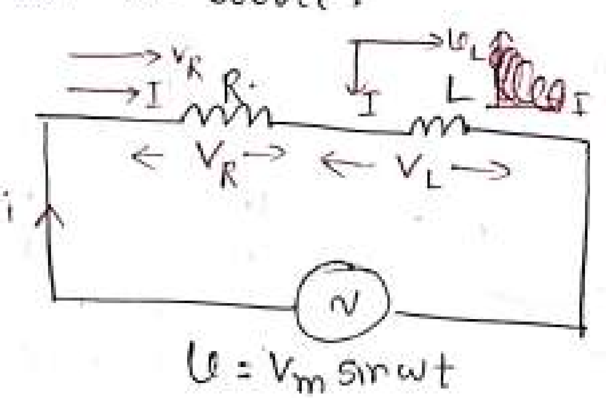
$\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

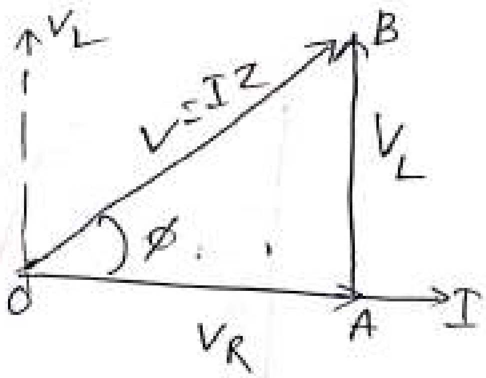
a + jb

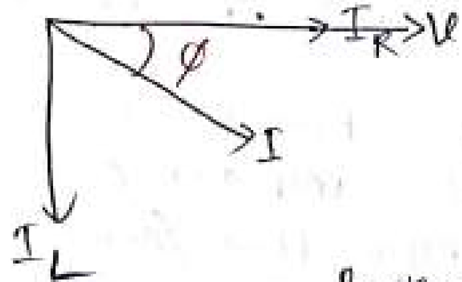


AC through Resistance & inductance

A pure resistance R and a pure inductive coil of inductance L are connected in series with an AC source.







in R-L series circuit, current lags behind Voltage by an angle ϕ.

Power factor = $\cos\phi$

The applied Voltage V is the vector sum of the two i.e. OB.

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{(R^2 + X_L^2)}$$

or $I = \dfrac{V}{\sqrt{R^2 + X_L^2}} = \dfrac{V}{Z}$

$Z = \sqrt{R^2 + X_L^2}$ = impedance of the circuit.

where R = Resistance of the circuit

X_L = inductive reactance of the circuit $= 2\pi f L = \omega L$

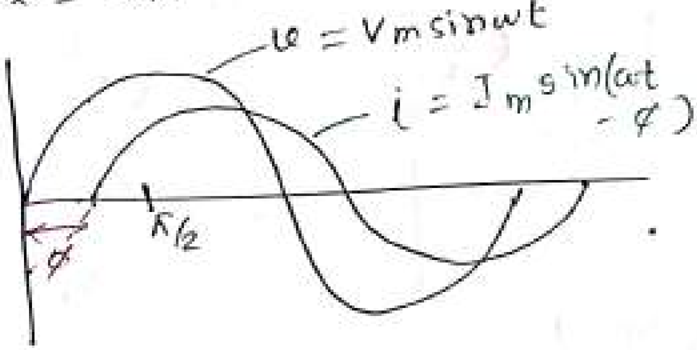
$\omega = 2\pi f$

unit Z	= ohm
R	= ohm
X_L	= ohm
L	= Henry

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R} \quad \boxed{\phi = \tan^{-1}\left(\frac{X_L}{R}\right)}$$

if $v = V_m \sin\omega t$

$i = I_m \sin(\omega t - \phi)$ where $I_m = \dfrac{V_m}{Z}$



ϕ = Phase angle.

In series R-L ckt, the current lags behind the voltage by an angle 'ϕ'.

Power Factor

→ It is defined as cosine of the angle b/w v and i.

(or) → the ratio of $\dfrac{R}{Z} = \dfrac{\text{Resistance}}{\text{impedance}}$

→ the ratio of $\dfrac{\text{true power/Active Power}}{\text{Apparent power}} = \dfrac{VI\cos\phi}{VI}$

$= \cos\phi$

Power ~~triangle~~ in AC circuit

Active power (P)

→ The product of voltage (Rms) and current (Rms) with the cosine of the angle b/w them in an AC circuit is termed as active power.

→ Active component of power lies in phase with the applied voltage.

→ unit = Watts (W)

$$\boxed{\text{Active power } P = VI\cos\phi}$$

Reactive power (Q)

→ The product of voltage (Rms) & current (Rms) with sine of the angle b/w them in AC circuit is known as reactive power.

→ unit = Volt-Ampere reactive (VAR)

$$\boxed{Q = VI\sin\phi}$$

Apparent power (S)

→ The product of voltage (Rms) and current (Rms) is called as the apparent power.

→ unit = Volt-Ampere (VA)

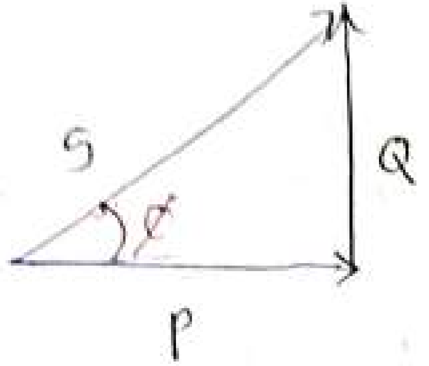
$$\boxed{S = VI}$$

$$\boxed{S^2 = P^2 + Q^2 \quad \text{or} \quad S = \sqrt{P^2 + Q^2}}$$

Power triangle

→ It is the representation of a right angle triangle showing the relation b/w Active power (P), reactive power (Q) and apparent power (S).

Power triangle in R-L series circuit



$Z = R + jX_L$

multiply I in both sides

$IZ = IR + jIX_L$

$V = IR + jIX_L$

multiply I

$VI = I^2R + jI^2X_L$

$$\boxed{S = P + jQ_L}$$

S = Total or Apparent (VA) power

P = Active power / Real power / true power (Watts)

Q = Reactive power (VARs)

$|S| = \sqrt{P^2 + Q_L^2}$

$\phi = \tan^{-1}\left(\frac{Q_L}{P}\right)$

Power Factor

$\cos\phi = \frac{P}{S}$

$P = S\cos\phi = VI\cos\phi$ Watt

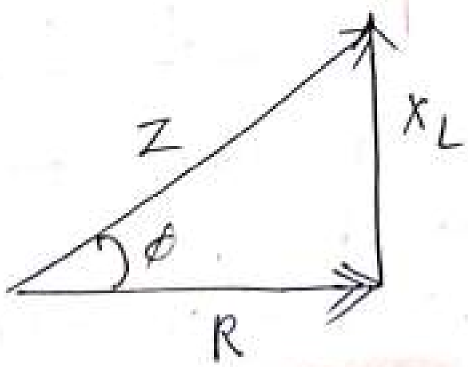
$Q_L = S\sin\phi = VI\sin\phi$ VAR's

Impedance triangle in R-L series circuit

$Z = R + jX_L \quad |Z| = \sqrt{R^2 + X_L^2}$

$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

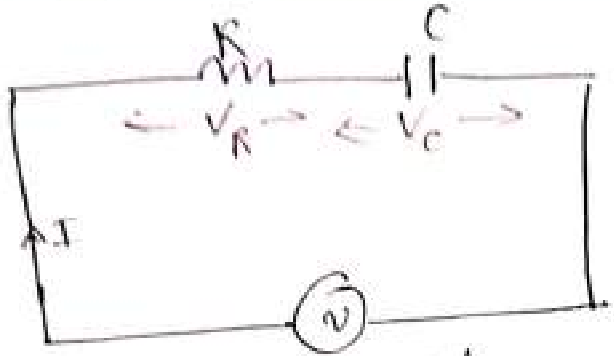
$= \tan^{-1}\left(\frac{\omega L}{R}\right)$



Power Factor: $\cos\phi = \frac{R}{Z}$

$\sin\phi = \frac{X_L}{Z}$

Ac through Resistance & capacitance



X_C = capacitive reactance

$= \frac{1}{\omega C} = \frac{1}{2\pi f C}$ ohm

$\boxed{f = \frac{1}{T}}$

$v = V_m \sin\omega t$

Here $V_R = IR$ = drop across 'R' in phase with 'I'.

$V_C = IX_C$ = drop across capacitor lag current by $\pi/2$.

$$V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

or $\boxed{I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}}$

$$\boxed{\begin{aligned} Z &= R - jX_C \ \Omega \\ Z &= \sqrt{R^2 + (-X_C)^2} = \sqrt{R^2 + X_C^2}\ \Omega \end{aligned}}$$

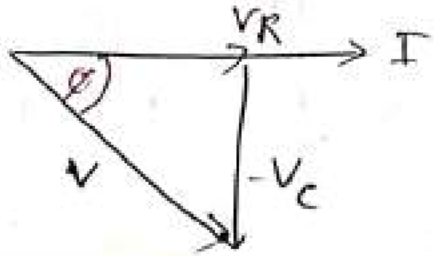
where $Z = \sqrt{R^2 + X_C^2}$ = impedance of the circuit

unit

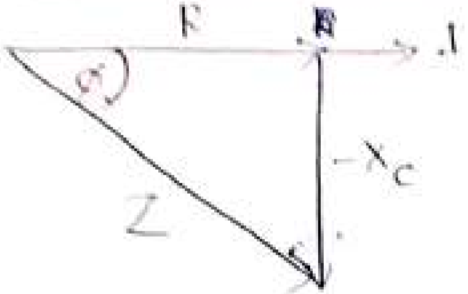
R = ohm	ω = rad/sec
C = Farad	f = Hz
X_C = ohm	$\omega = 2\pi f$
Z = ohm	$= \frac{2\pi}{T}$

As capacitive reactance X_C is taken negative, V_C is shown along negative direction of Y-axis in the voltage triangle.

Voltage triangle



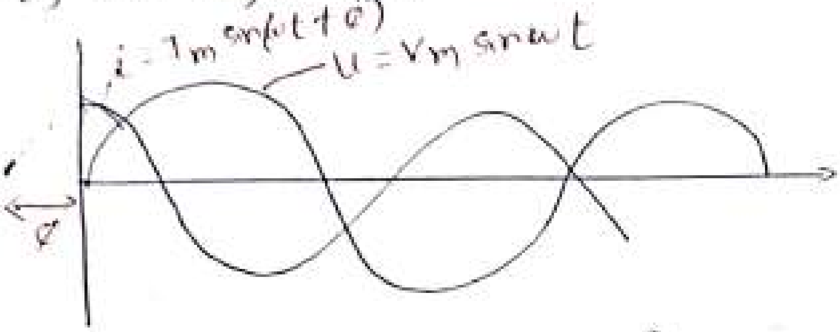
Impedance triangle



$\cos\phi = \frac{R}{Z}$

$\tan\phi = \frac{-X_C}{R}$

In R-C series circuit, the current leads voltage by an angle ϕ.



Power factor $\cos\phi = \frac{R}{Z}$, $\sin\phi = \frac{X_C}{Z}$

$Z = R - jX_C$

$IZ = RI - jX_C I$ (multiplying 'I' on both sides)

$IZ = IR - jIX_C$ (multiplying 'I' on both sides)

$V = IR - jIX_C$

$VI = I^2R - jI^2X_C$

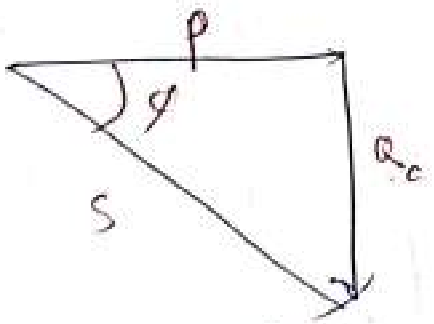
$S = P - jQ_c$

S = Apparent Power / Total Power = unit (VA)

P = Real/Active / True Power unit (Watt)

Q = Reactive Power (VAR)

Power triangle



$|S| = \sqrt{P^2 + Q_c^2}$

$\phi = \tan^{-1}\left[\frac{Q_c}{P}\right]$

Power factor $\cos\phi = \frac{P}{S}$, $\sin\phi = \frac{Q_c}{S}$

$\Rightarrow P = S\cos\phi$, $Q_c = S\sin\phi$

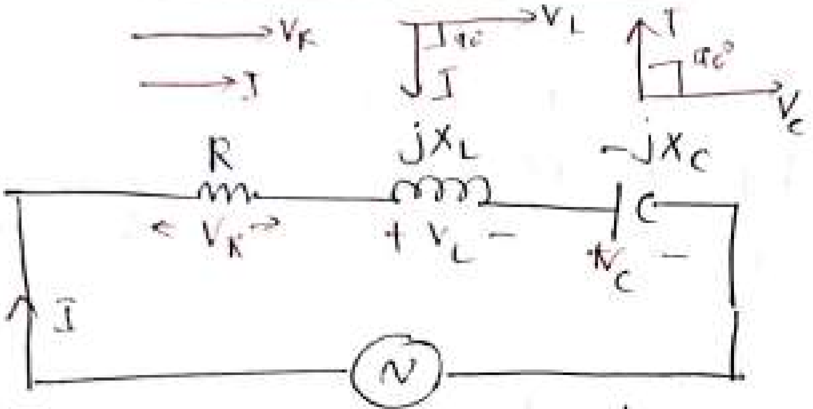
$\boxed{P = VI\cos\phi}$, $\boxed{Q_c = VI\sin\phi}$

load point of view

R-L $+Q_L \rightarrow$ (absorbing VAR's / lagging VAR's)

R-C $-Q_C \rightarrow$ (Generating VAR's / leading VAR's)

Series R, L, C circuit



$v = V_m \sin\omega t$

$V_R = IR$ = voltage drop across 'R' in phase with 'I'

$V_L = IX_L$ = voltage drop across 'L' lead 90° with 'I'

$V_C = IX_C$ = voltage drop across 'C' lag 90° from 'I'

here $\vec{V} = \vec{V_R} + \vec{V_L} + \vec{V_C}$

$Z = R + j(X_L - X_C) \Rightarrow Z = R \pm j X_{net}$ Ω

$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$ Ω

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

$\hookrightarrow$ net impedance angle

Power Factor, $\cos\phi = \frac{R}{Z}$, $\sin\phi = \frac{X_{net}}{|Z|}$

$= \frac{(X_L - X_C)}{|Z|}$

$S = P + j(Q_L - Q_C)$

$|S| = \sqrt{P^2 + (Q_L - Q_C)^2}$

$\phi_{net} = \tan^{-1}\left(\frac{Q_L - Q_C}{P}\right)$

Power Factor $\cos\phi = \frac{P}{S}$ $\Rightarrow P = S\cos\phi$

$P = VI\cos\phi$

$\sin\phi = \frac{Q_{net}}{S} = \frac{(Q_L - Q_C)}{S}$, $Q_{net} = S\sin\phi = VI\sin\phi$.

case-i $\boxed{\text{if } X_L > X_C}$

$Z = R + j(X_L - X_C)$

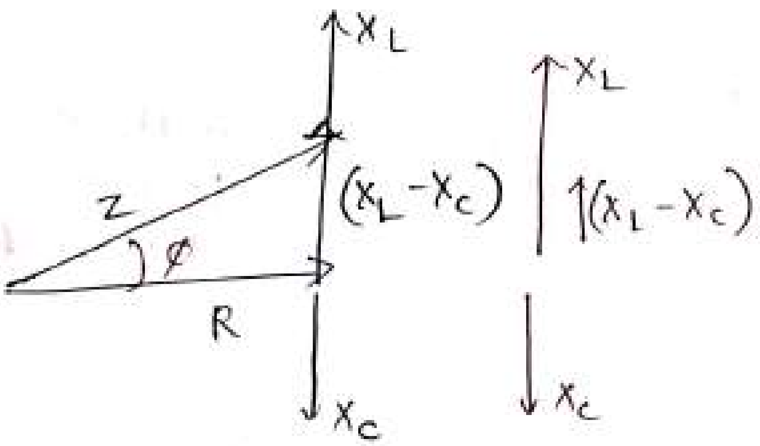
$Z = R + jX_{net}$

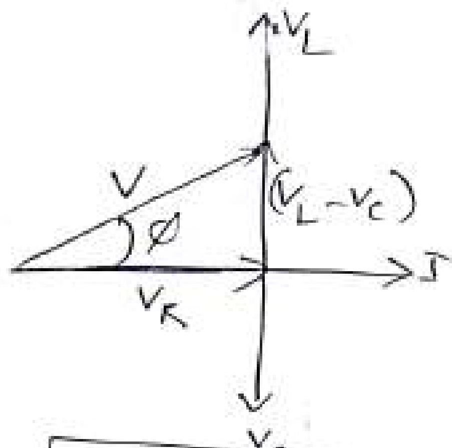
$\hookrightarrow$ behaves as R-L series circuit

$\rightarrow$ 'I' lag's 'V' by $\phi < 90^\circ$

$\rightarrow$ lagging power factor

$(V_L > V_C)$





$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$

case-ii $\boxed{\text{if } (X_L < X_C)}$

$Z = R + j(X_L - X_C)$

$Z = R - jX_{net}$

$\longrightarrow$ behaves as R-C series ckt.

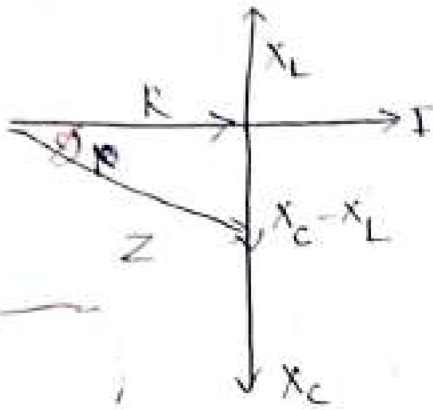
$\rightarrow$ 'I' leads 'V' by $\phi < 90^\circ$.

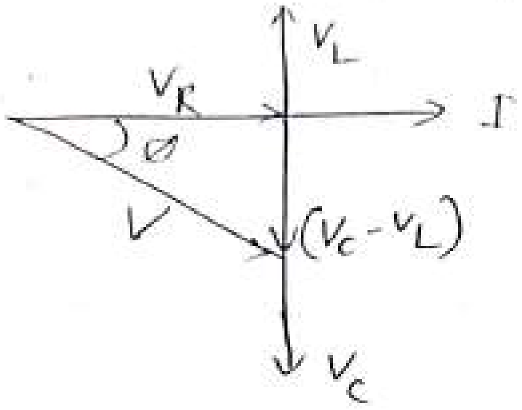
$\rightarrow$ leading power factor

$V_C > V_L$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$





$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

$$\phi = \tan^{-1}\left(\frac{V_C - V_L}{V_R}\right)$$

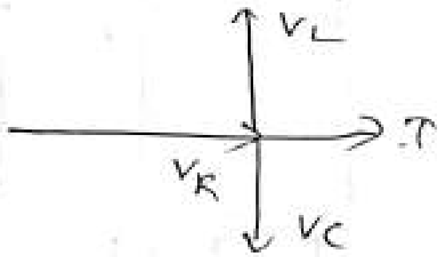
ϕ = Phase angle or impedance angle.

<u>Case-III</u> $\boxed{\text{if } X_L = X_C}$, $V_L = V_C$

$Z = R$, circuit behaves as purely resistive circuit.

$\phi = 0^\circ$

$\cos\phi = 1$



$Z = R$

circuit behaves as purely resistive circuit.

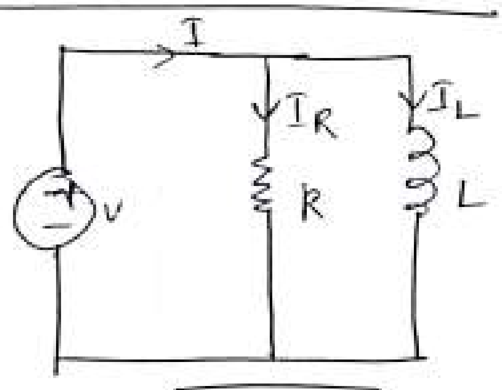
→ power factor UPF (unity power factor)

→ This is called resonance.

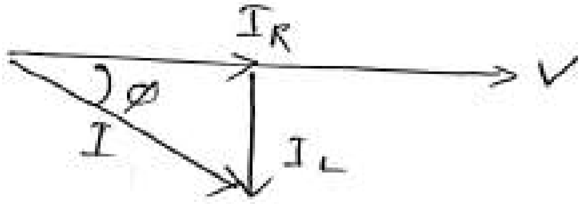
Summary of series AC circuits

	Impedance (Z)	Phase angle (ϕ)	Power factor
pure 'R'	R	0°	1
pure 'L'	$X_L = \omega L = 2\pi f L$	90° lag	0
pure 'C'	$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f L}$	90° lead	0
R-L series	$Z = \sqrt{R^2 + X_L^2}$	$0 < \phi < 90^\circ$ lag	$1 > pf > 0$ lag
R-C series	$Z = \sqrt{R^2 + X_C^2}$	$0 < \phi < 90^\circ$ lead	$1 > pf > 0$ lead
R-L-C	$Z = \sqrt{R^2 + (X_L - X_C)^2}$	$0^\circ < \phi < 90^\circ$ lead or lag	$1 > pf > 0$ lead or lag.

Parallel R-L circuit



$I_R = \frac{V}{R}$, $I_L = \frac{V}{Z_L} =$

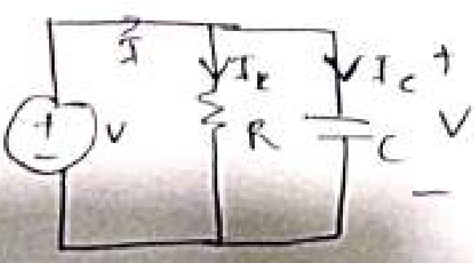


$I = \sqrt{I_R^2 + I_L^2}$

$\phi = \tan^{-1}\left(\frac{I_L}{I_R}\right)$, $\cos\phi = pf = \frac{I_R}{I}$ (lag)

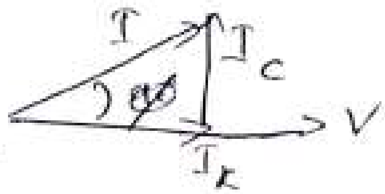
'I' lags 'V' by 'ϕ'.

Parallel R-C circuit



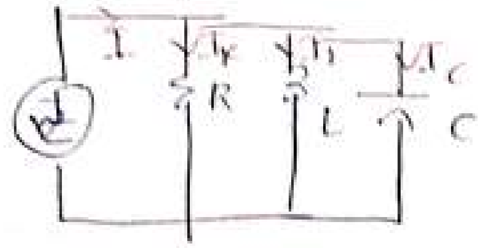
$I_R = \frac{V}{R}$, $I_C = \frac{V}{Z_C}$

'I' leads 'V' by 'ϕ'

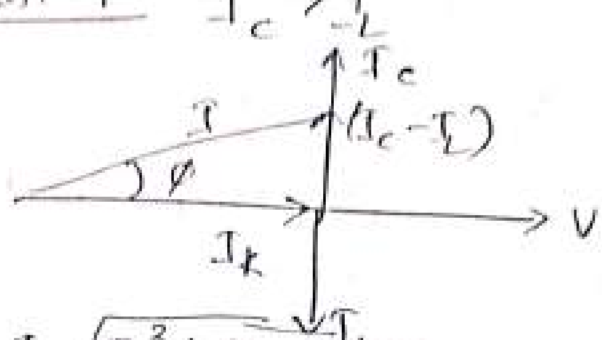


$I = \sqrt{I_R^2 + I_C^2}$

Parallel R-L-C circuit



Case-I $I_C > I_L$



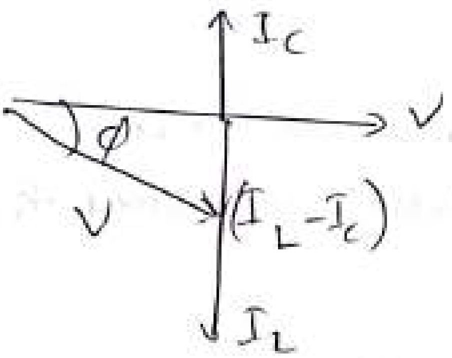
$\cos\phi = P.F = \frac{I_R}{I}$ (lead)

'I' leads 'V'.

$I = \sqrt{I_R^2 + (I_C - I_L)^2}$

$\phi = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right)$

Case-II $(I_L > I_C)$



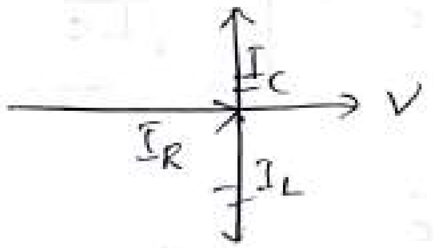
$I = \sqrt{I_R^2 + (I_L - I_C)^2}$

$\phi = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right)$

$\cos\phi = P.f = \frac{I_R}{I}$ (lag)

'I' lags 'V'.

Case-III $I_L = I_C$



$\phi = 0^\circ$ $I = I_R$

↳ unity Power Factor

→ the circuit is under resonance.

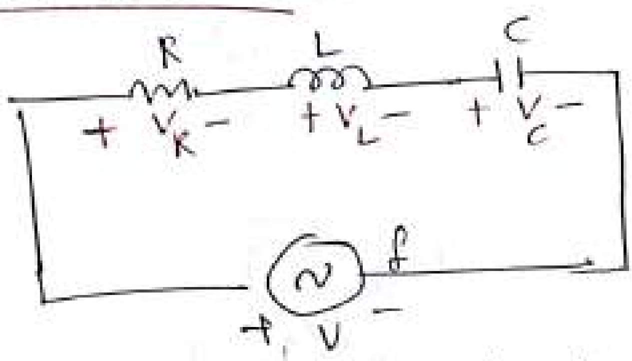
Resonance in R-L-C circuits

When X_L equals to X_C in magnitude, in that case

$$X_{net} = X_L - X_C = 0$$

$Z = \sqrt{R^2 + X_{net}^2} = R$ the circuit is under electrical resonance.

Series Resonance



The frequency at which the net reactance of the series R-L-C series circuit is called the the resonant frequency(f_0).

$\Rightarrow X_L - X_C = 0$

$\Rightarrow X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$ or $\omega^2 = \frac{1}{LC}$

$\Rightarrow 2\pi f L = \frac{1}{2\pi f C} \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$

$\Rightarrow 4\pi^2 f^2 LC = 1$

$\Rightarrow f^2 = \frac{1}{4\pi^2 LC}$

$\Rightarrow \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}}$

Unit

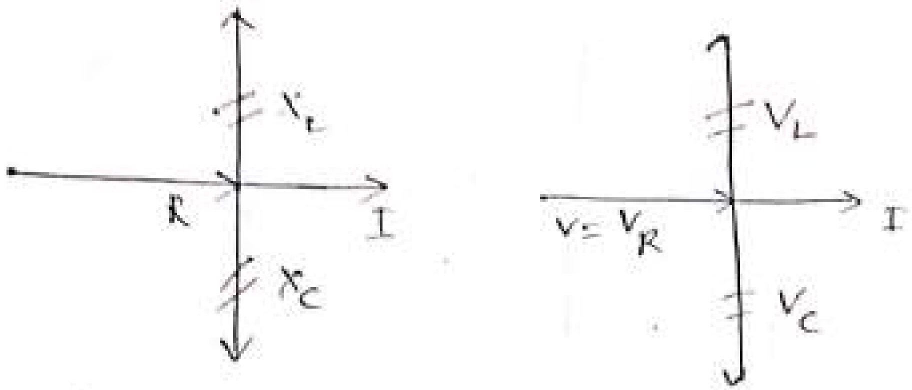
$L \rightarrow$ Henry

$C \rightarrow$ Farad

$f_0 \rightarrow$ Hz

$\omega_0 \rightarrow$ rad/sec

Phasor diagram under resonance condition



Summary

When a R-L-C circuit is in resonance

i) Net reactance of the circuit is zero.
i.e. $X_L - X_C = 0 \Rightarrow \boxed{X_{net} = 0}$

ii) circuit impedance is minimum i.e. $\boxed{Z_0 = R}$. consequently circuit admittance is maximum.

iii) circuit ~~impedance~~ current is ~~minimum~~ maximum & is given by $\boxed{I_0 = \frac{V}{Z_0} = \frac{V}{R}}$

iv) Power dissipated is maximum i.e. $\boxed{P_0 = I_0^2 R = \frac{V^2}{R}}$

v) circuit power factor angle $\boxed{\theta = 0^\circ.}$
Power Factor $\boxed{\cos\phi = 1}$

vi) Although $V_L = V_C$ yet V_{coil} is greater than V_C because of its resistance.

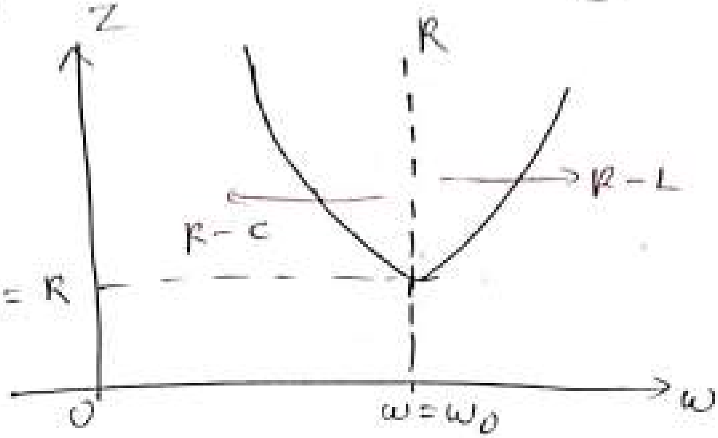
vii) At resonance, $\omega^2 LC = 1$.

viii) Q Factor $Q = \tan\theta = \tan 0^\circ = 0^\circ$

→ Since circuit current is maximum, it produces large voltage drop across L and C.

Graph - I $\boxed{\text{'Z' vs 'ω'}}$

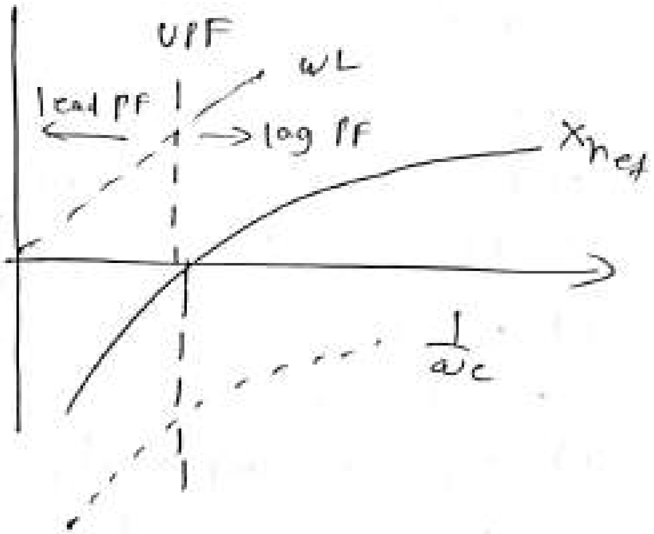
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



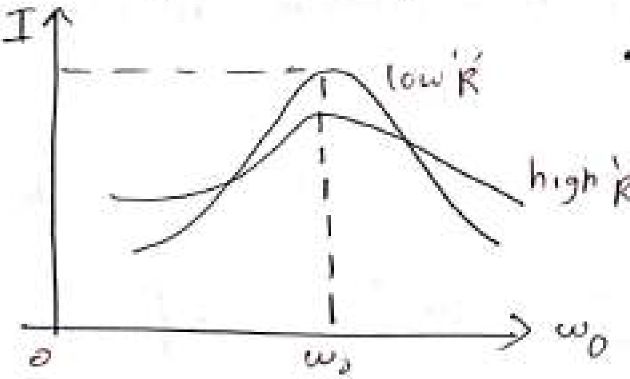
1.

Graph - II $\boxed{\text{'X' vs 'ω'}}$

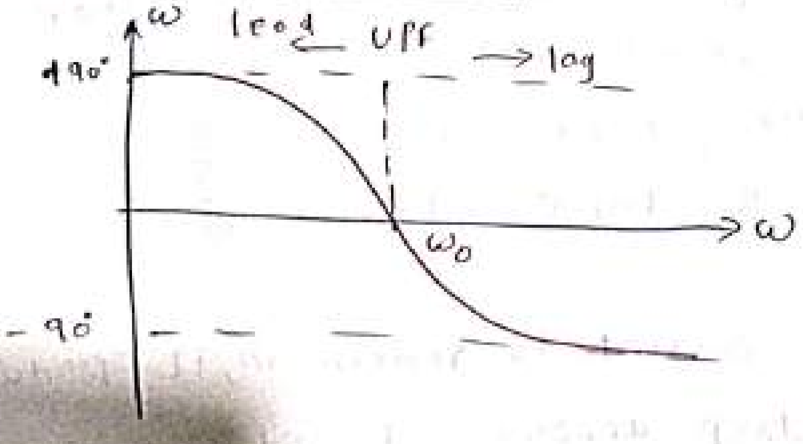
$$X_{net} = \left(\omega L - \frac{1}{\omega C}\right)$$



Graph - III $\boxed{\text{I vs 'ω'}}$



Graph - IV $\boxed{\text{∅ vs 'ω'}}$



→ At resonance, impedance 'Z' is minimum.
current 'I' is maximum.

→ Hence it is sometimes called as acceptor circuit.

At $\omega = \omega_0$ or $X_L = X_C$ or $f = f_0$

$X_{net} = 0$, so $I_0 = \frac{V}{R}$

so Power is maximum $P_0 = I_0^2 R$

At $\omega = \omega_0$, $\boxed{R} \Rightarrow V_R = I_R R = I_0 R = \frac{V}{R} \cdot R = V$

$\Rightarrow \boxed{V_R = V}$

$\boxed{L} \Rightarrow V_L = +jI_L X_L = +jI_L \omega_0 L$

$= +j\left(\frac{\omega_0 L}{R}\right) V.$

$\boxed{V_L = +j\, Q_0 V}$

$\boxed{C} \Rightarrow V_C = -jI_C X_C = \frac{-j}{\omega_0 C} I_0 = -j\left[\frac{1}{\omega_0 RC}\right] V$

$\boxed{V_C = -j Q_0 V}$

Q_0 = Voltage amplification factor

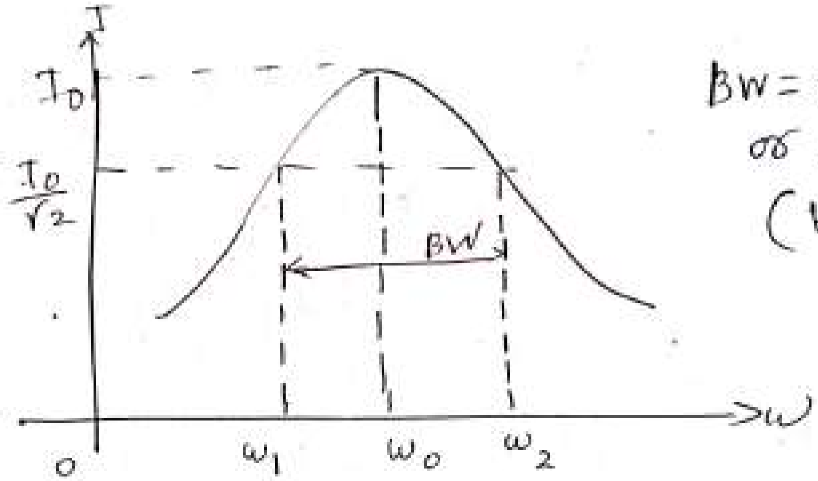
⇒ At series resonance condition, as if the total Voltage appears across resistor only, hence it is called voltage amplification circuit.

→ The series resonance phenomena is used in designing receivers, antennas in communication engineering with passive components.

Bandwidth

Bandwidth represents the range of frequencies for which power level in signal is atleast $\frac{1}{2}$ of maxm power.

(or) The bandwidth of any system is the range of frequencies for which the current or o/p voltage is equal to 70.7% of its value at the resonant frequency.



$BW = \omega_2 - \omega_1$
or $f_2 - f_1$
(Hz)

ω_1 = lower cutoff / corner frequency
ω_2 = upper cut-off / corner frequency.

$$P = \frac{P_0}{2} = \frac{I_0^2 R}{2} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = (0.707\, I_0)^2 R$$

At $\omega = \omega_1$, $\left[\omega_1 L - \frac{1}{\omega_1 C}\right] = -R$

At $\omega = \omega_2$ $\left[\omega_2 L - \frac{1}{\omega_2 C}\right] = +R$

$$\omega_0^2 = \frac{1}{LC} \Rightarrow \boxed{\omega_0 = \sqrt{\omega_1 \omega_2} \text{ or } f_0 = \sqrt{f_1 f_2}}$$

Bandwidth

$BW = \omega_2 - \omega_1 = \frac{R}{L}$ rad/sec

or $= f_2 - f_1 = \frac{R}{2\pi L}$ Hz

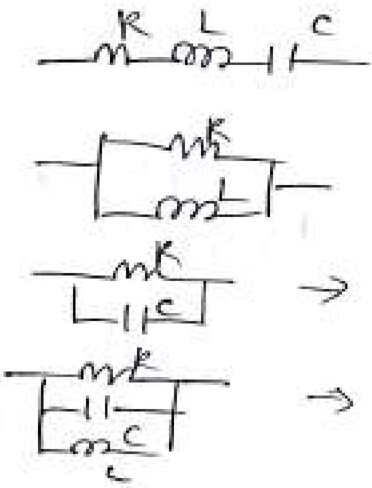
$\boxed{\because BW \propto R}$ but f_0 is independent of 'R'

Quality factor (Q-Factor) / Figure of merit (23)

The Quality Factor 'Q' is the ratio of the reactive power in the inductor or capacitor to the true power in the resistance in series with the coil or capacitor.

$$Q = 2\pi \times \frac{\text{max}^m \text{ energy stored}}{\text{energy dissipated per cycle}}$$

element	Q-Factor
R	0
L	∞
C	∞
R L	$\frac{\omega L}{R}$
R C	$\frac{1}{\omega RC}$



$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$

$Q_0 = \frac{R}{\omega L}$

$\rightarrow$ $Q = \omega RC$

$\rightarrow$ $Q = R\sqrt{\frac{C}{L}}$

Selectivity (S)

It is the ability of a Network to discriminate or distinguish b/w desired & undesired frequency.

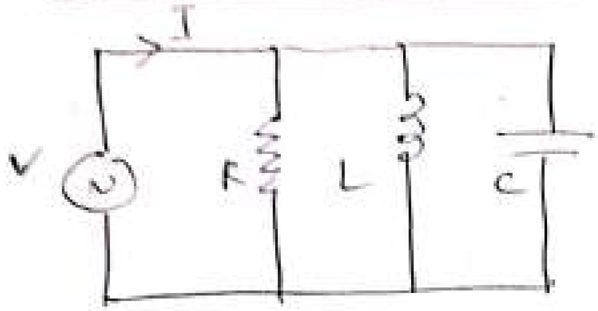
$$S = \frac{f_0}{|BW|} = \frac{f_0}{|f_2 - f_1|} = \frac{1/2\pi\sqrt{LC}}{R/2\pi L}$$

$$\boxed{S = \frac{1}{R}\sqrt{\frac{L}{C}} = Q_0}$$

$$\boxed{S \propto \frac{1}{|BW|}}$$

Parallel resonance

(a) R-L-C Parallel resonance circuit



Admittance $(Y) = G + jB$

$$Y = Y_R + Y_L + Y_C$$

$$= \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

The frequency at which resonance occurs

net succeptance $= 0$

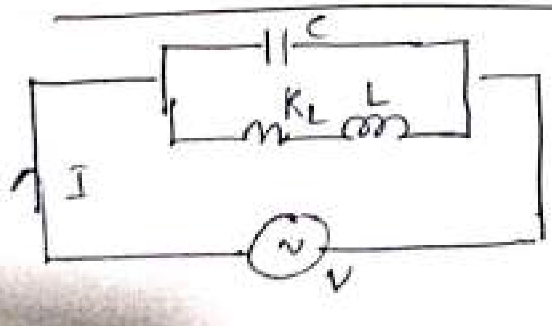
$$B_{net} = 0 \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec} \qquad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}}$$

during parallel resonance the net impedance is max^m, so current is minimum. Hence it is called rejector circuit.

$$BW = \omega_2 - \omega_1 = \frac{1}{RC}$$

$$Q\text{-Factor} = \omega RC$$

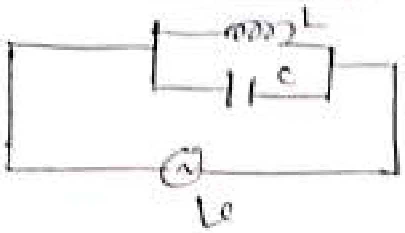
(b) Practical parallel resonance (Tank circuit)



$$Y = Y_1 + Y_2 = \frac{1}{R + jX_L} + \frac{1}{-jX_C}$$

$$\boxed{\omega_0 = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} \text{ rad/sec} \qquad f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} \text{ Hz}}$$

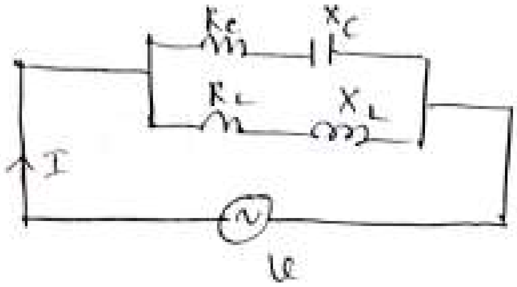
(c) Ideal tank circuit



$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

(d)

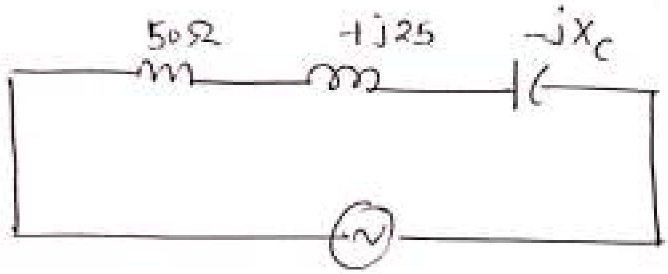


$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - (L/C)}{R_C^2 - (L/C)}}$$

if $R_L = R_C$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec} \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

Q)



For the circuit shown determine the value of capacitive reactance & impedance at resonance.

A: At resonance condition,

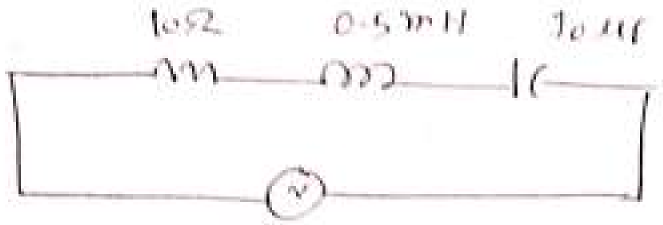
$$\boxed{X_L = X_C} \qquad X_L = 25\Omega \text{ given}$$

then $X_C = 25\Omega$

$$\Rightarrow \frac{1}{\omega C} = 25$$

$\therefore$ The value of impedance at resonance is

$$Z = R \quad \boxed{\therefore Z = 50\Omega}$$

Q) 

(26)

Determine resonant frequency for the circuit shown?

A: $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$L = 0.5\,mH$
$= 0.5 \times 10^{-3}\,H$

$C = 10\,\mu F = 10 \times 10^{-6}\,F$

$$f_0 = \frac{1}{2\pi\sqrt{10 \times 10^{-6} \times 0.5 \times 10^{-3}}}$$

$$\boxed{f_0 = 2.25\,KHZ}$$ (Ans)

Polyphase Circuits

Polyphase → more than one phase

→ In a 2-phase alternator, the armature windings are displaced 90° electrical degrees apart.

→ In a 3-ϕ alternator, the armature windings which are 120° electrical degrees apart.

Electrical displacement b/w different phases is

$= \frac{360°}{n}$ where n = no. of phases or windings (except 2 phase).

Advantages of '3' phase system

i) It is more efficient

ii) It uses less material for a given capacity

iii) It costs less than single phase apparatus.

iv) In a given frame size, a 3-ϕ motor or a 3-ϕ generator produces more o/p than its single phase counterpart.

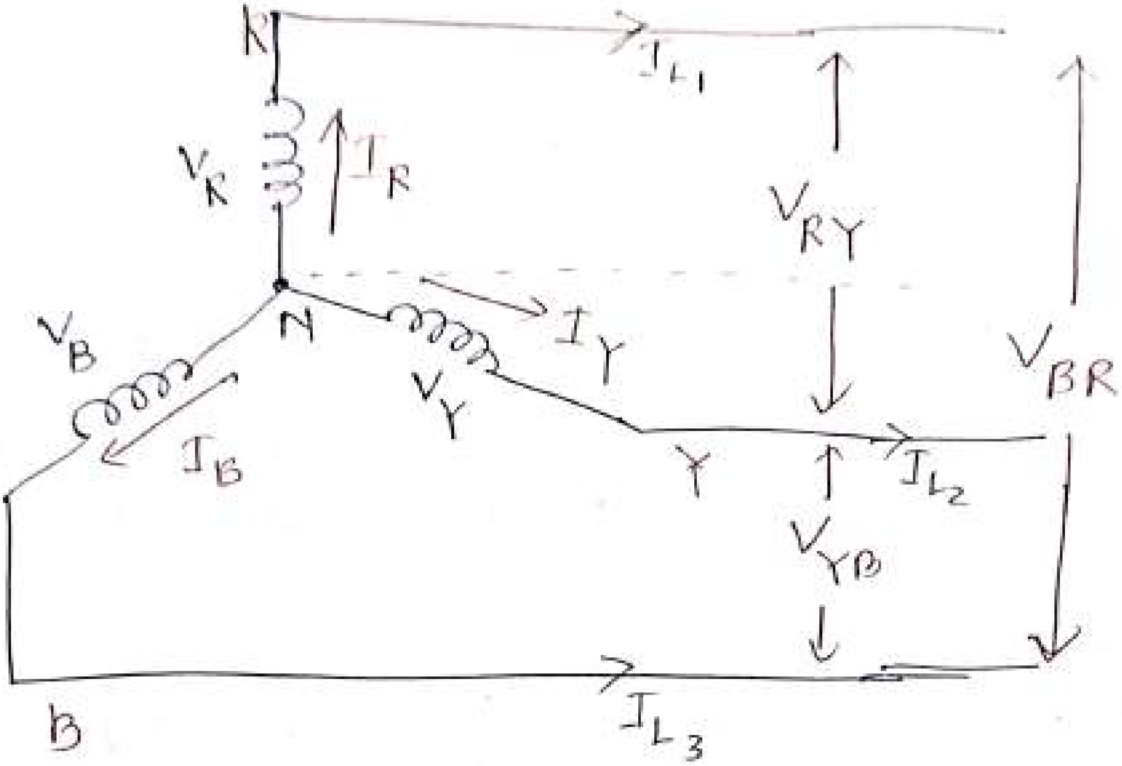
v) Three-phase motors are more easily started than 1-ϕ motors. 1-ϕ motors are not self starting but 3-ϕ motors are.

Phase sequence

→ By phase sequence is meant the order in which the '3' phases attain their peak or maximum values.

→ $a \rightarrow b \rightarrow c$ is called phase sequence or phase order.

Star connected Balanced 3-ϕ supply



$$I_{L_1} = I_{L_2} = I_{L_3} = I_R = I_Y = I_B$$

i.e. $\boxed{I_{Ph} = I_L}$ in Y-connection

$$V_{RN} = V_{BN} = V_{YN} = V_{Ph} \text{ (phase voltage)}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L \text{ (line voltage)}$$

$$\boxed{\begin{aligned} V_{RY} &= V_R - V_Y \\ V_{YB} &= V_Y - V_B \\ V_{BR} &= V_B - V_R \end{aligned}}$$

→ The voltage induced in each winding is called the phase voltage & current in each winding is likewise known as phase current.

→ The voltage available between any two phases is known as line voltage (V_L) & the current flowing in any line is known as line current.

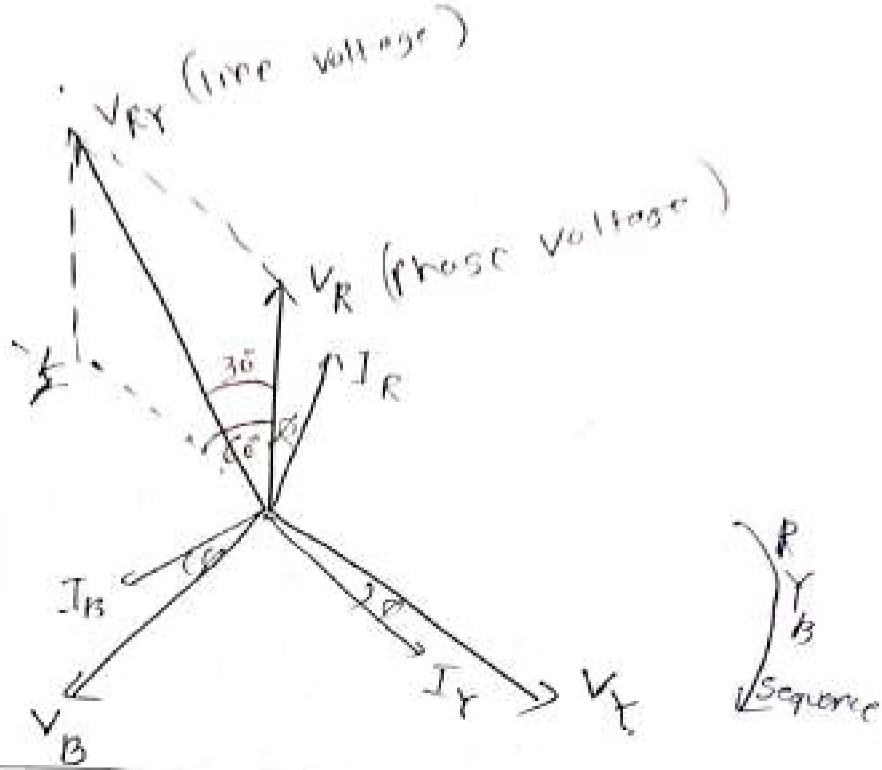
→ The voltage available b/w any one line & Neutral is known as phase voltage.

Phase currents lag to phase voltages by an angle of ϕ.

Phasor diagram considering phase voltages as reference

→ For lagging load

$$\boxed{\begin{aligned} V_R = V_Y = V_B &= V_{ph} \\ V_{RY} = V_{YB} = V_{BR} &= V_L \end{aligned}}$$



$$V_{RY} = \sqrt{V_R^2 + (-V_Y)^2 + 2V_R(-V_Y)\cos 60°}$$

$$= \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}^2 \times \frac{1}{2}}$$

$$= \sqrt{2V_{ph}^2 + V_{ph}^2}$$

$$V_L = \sqrt{3V_{ph}^2} \qquad \therefore \boxed{V_L = \sqrt{3}\, V_{ph}}$$

From Fig (summary)

1. line voltages are 120° apart.
2. line voltages are 30° ahead of their respective phase voltages.
3. The angle between the line currents and the corresponding line voltages is $(30° + \phi)$ with current lagging.

$$\boxed{\begin{aligned} I_L &= I_{ph} \\ V_L &= \sqrt{3}\, V_{ph} \end{aligned}}$$

Power The total active or true power in the circuit is the sum of 3-Phase powers.

Total Active power = 3X Phase power

$$P = 3 \times V_{ph} I_{ph} \cos\phi$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} \quad \text{and } I_{ph} = I_L$$

In terms of line voltages

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\phi \quad \boxed{P = \sqrt{3}\, V_L I_L \cos\phi \text{ watt}}$$

Total reactive power, $\boxed{Q = \sqrt{3}\, V_L I_L \sin\phi \text{ VAR}}$

$$\boxed{S = \sqrt{3}\, V_L I_L} \quad \boxed{S = \sqrt{P^2 + Q^2}}$$

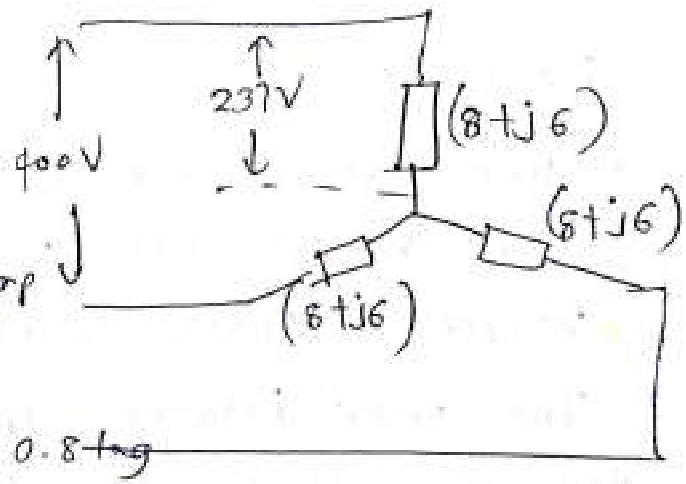
VAs

a) A balanced star-connected load of $(8+j6)\,\Omega$ per phase is connected to a balanced 3-Phase, 400V-supply. Find the line current, power factor & total volt-amperes.

A: $Z_{ph} = \sqrt{8^2+6^2} = 10\,\Omega$

$$V_{ph} = \frac{400}{\sqrt{3}} = 231\text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{10} = 23.1\text{ Amp}$$



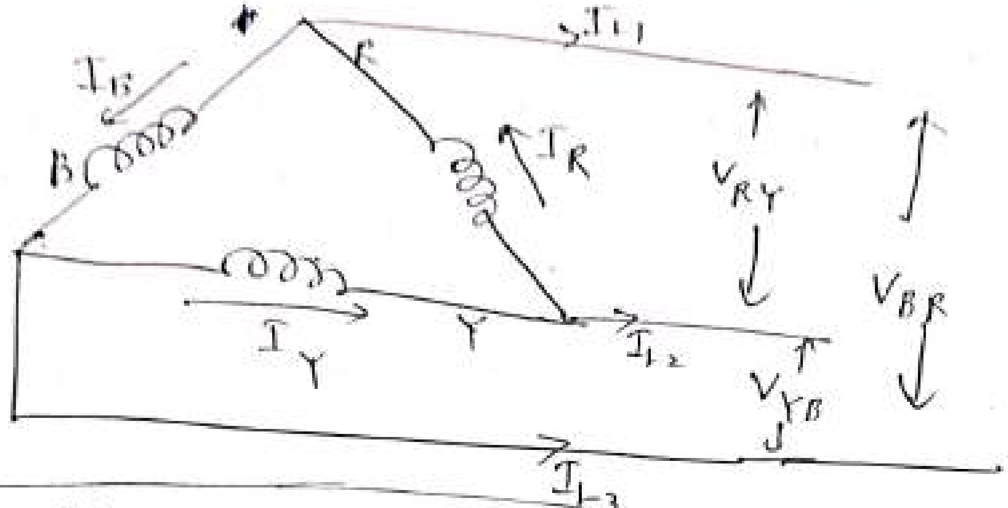
i) $I_L = I_{ph} = 23.1$ Amp

ii) $PF = \cos\phi = \frac{R}{Z} = \frac{8}{10} = 0.8$ ~~lag~~ lag

iii) power $= \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 23.1 \times 0.8 = 12{,}800$ W

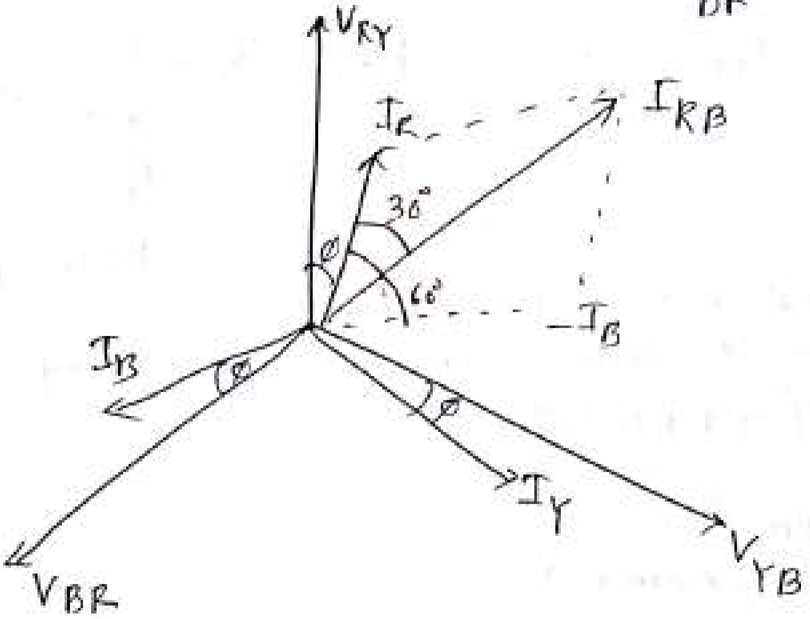
iv) $S = \sqrt{3}\, V_L I_L = \sqrt{3} \times 400 \times 23.1 = 16000$ VA.

Delta connected Balanced three phase supply



$$\boxed{V_{RY} = V_{YB} = V_{BY} = V_L = V_{ph}}$$

$$I_{RY} = I_R - I_Y$$
$$I_{YB} = I_Y - I_B$$
$$I_{BR} = I_B - I_R$$



$$I_R = I_Y = I_B = I_{ph}$$
$$I_{RY} = I_{YB} = I_{BR} = I_L$$

$$I_{RB} = \sqrt{I_R^2 + (-I_B)^2 + 2 I_R I_B \cos 60^\circ}$$
$$= \sqrt{I_{ph}^2 + I_{ph}^2 + 2 I_{ph}^2 \times \frac{1}{2}}$$
$$I_{RB} = \sqrt{3 I_{ph}^2} \qquad \boxed{I_L = \sqrt{3}\, I_{ph}}$$

Summary for delta connection

1. $V_L = V_{ph}$
2. $I_L = \sqrt{3}\, I_{ph}$
3. I_L lags I_{ph} by $30°$. (line currents lags behind $30°$ to the respective phase current)
4. line currents are $120°$ apart.
5. Angle b/w the line currents & the corresponding line voltages is $(30+\phi)$ with the current lagging.

Power in delta connection

Power / Phase $= V_{ph} I_{ph} \cos\phi$

Total power $= 3 \times V_{ph} I_{ph} \cos\phi$

$\boxed{V_{ph} = V_L \quad \& \quad I_{ph} = \frac{I_L}{\sqrt{3}}}$

$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos\phi$

$\boxed{P = \sqrt{3}\, V_L I_L \cos\phi}$

ϕ = Phase power factor angle

Q) A 220V, 3-ϕ voltage is applied to a balanced delta connected 3-ϕ load of phase impedance $(15 + j20)\Omega$

i) Find I_{ph} ?

ii) power consumed ?

A: $V_{ph} = V_L = 220\,V$, $Z_{ph} = 15 + j20 = \sqrt{15^2 + 20^2} = 25\,\Omega$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{220}{25} = 8.8\,A$

a) $I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 8.8 = 15.24\,A$

b) $P = I_{ph}^2 R_{ph} = 8.8^2 \times 15 = 462\,W$

Summary for delta connection

1. $V_L = V_{ph}$
2. $I_L = \sqrt{3}\, I_{ph}$
3. I_L lags I_{ph} by $30°$. (line currents lags behind $30°$ to the respective phase current)
4. line currents are $120°$ apart.
5. Angle b/w the line currents & the corresponding line voltages is $(30+\phi)$ with the current lagging.

Power in delta connection

Power / Phase = $V_{ph} I_{ph} \cos\phi$

Total power = $3 \times V_{ph} I_{ph} \cos\phi$

$$\boxed{V_{ph} = V_L \quad \& \quad I_{ph} = \frac{I_L}{\sqrt{3}}}$$

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos\phi$$

$$\boxed{P = \sqrt{3}\, V_L I_L \cos\phi}$$

ϕ = Phase power factor angle

Q) A 220V, 3-ϕ voltage is applied to a balanced delta connected 3-ϕ load of phase impedance $(15 + j20)\,\Omega$

i) Find I_{ph} ?

ii) power consumed ?

A: $V_{ph} = V_L = 220\,V$, $Z_{ph} = 15 + j20 = \sqrt{15^2 + 20^2} = 25\,\Omega$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{220}{25} = 8.8\,A$$

a) $I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 8.8 = 15.24\,A$

b) $P = I_{ph}^2 R_{ph} = 8.8^2 \times 15 = 462\,W$

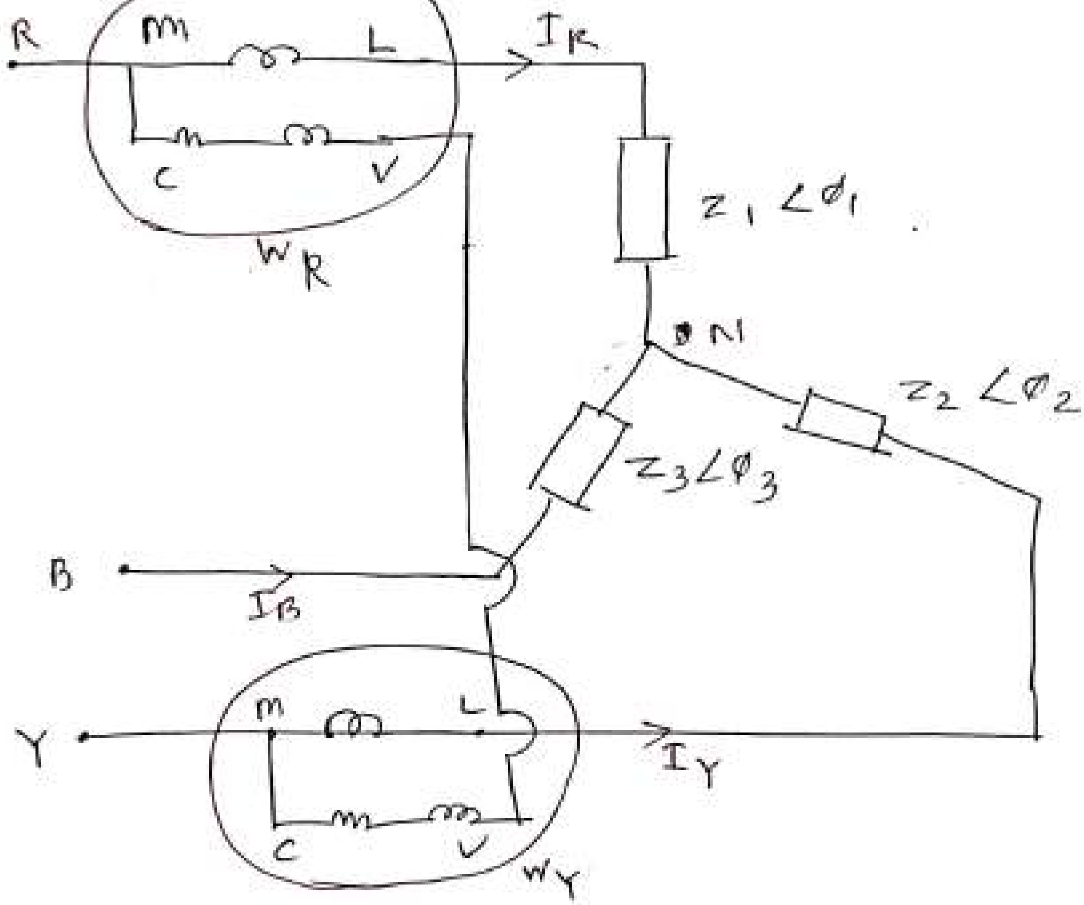
Measurement of 3-ϕ power

a) Three wattmeter method

In this method, '3' wattmeters are inserted one in each phase and the algebraic sum of their readings gives the total power consumed by 3-phase load.

b) Two-wattmeter method

3-phase, 2-wattmeter method



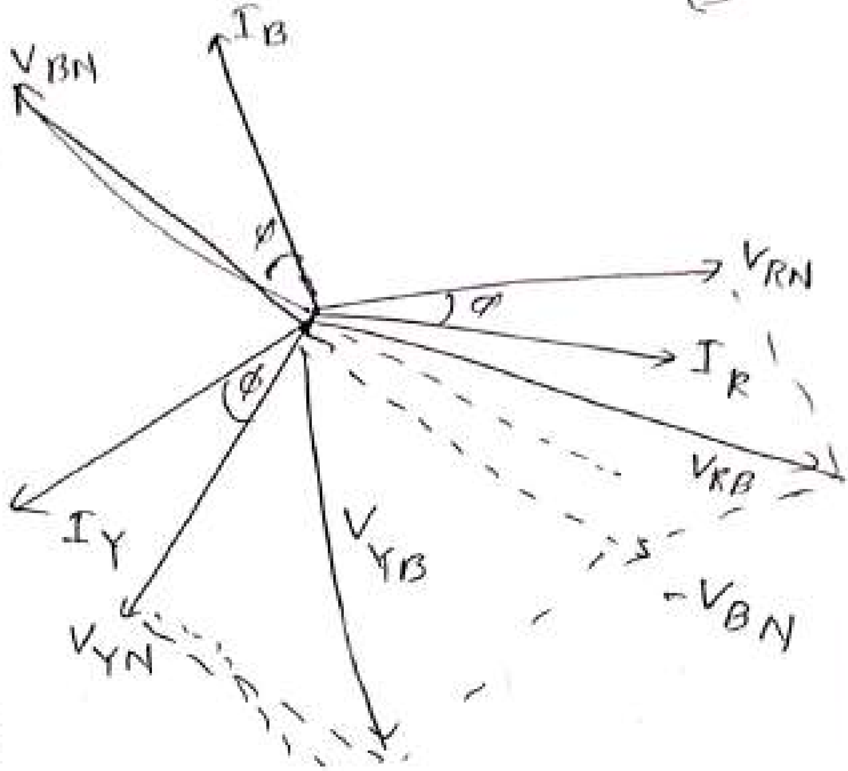
Total power $= W_R + W_Y$

Wattmeter W_R

i) current measured by current coil of wattmeter $- W_R = I_R$

ii) voltage measured by pressure coil of wattmeter $-$ ~~ϵ_d = ϵ_{RB}~~

$V_{RB} = V_R - V_B$



→ The Phase displacement b/w I_R & V_{RB} is equal to

$30° \rightarrow$ Resistive 1.
$(30° - \phi) \rightarrow$ inductive
$(30° + \phi) \rightarrow$ capacitive

$\boxed{\text{Wattmeter } W_Y}$

i) current measured by current coil of wattmeter $= I_Y$

ii) Voltage measured by pressure coil $= V_{YB} = V_Y - V_B$

(iii) the Phase displacement b/w I_y & V_{YB} is
$(30° + \phi) \rightarrow$ inductive
$30° \rightarrow$ Resistive
$(30° - \phi) \rightarrow$ capacitive.

$$\boxed{W_R = I_R V_{RB} \cos(30° - \phi)}$$

$$\boxed{W_Y = I_Y V_{YB} \cos(30° + \phi)}$$

For balanced load $I_R = I_Y = I_B = I_L$
$V_{RY} = V_{YB} = V_{BR} = V_L$

$$W_R + W_Y = V_L I_L \left[\cos(30° - \phi) + \cos(30° + \phi)\right]$$

So total active power

$$\boxed{P = W_R + W_Y = \sqrt{3}\, V_L I_L \cos\phi}$$

if P.F angle is 60°, one of the wattmeter reading is zero.

Case - (v) if $\phi = 90°$ $\quad W_1 = \frac{V_L I_L}{2}$, $W_2 = \frac{-V_L I_L}{2}$

$\Rightarrow \boxed{W_2 = -W_1}$

if P.F angle is 90°, one wattmeter reading is -ve.

Q) The two wattmeter method is used to measure power in a 3-ϕ load. The two wattmeter readings are 400W & -35W. calculate i) total active power
ii) power Factor
iii) reactive power

A:- $W_1 = 400W$ (higher wattmeter reading)
$W_2 = -35\,W$ (lower ~~a~~ wattmeter reading)

i) Total Active power $= W_1 + W_2 = 400 + (-35) = 365W$

ii) $\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} = \sqrt{3}\left[\frac{400-(-35)}{400-35}\right]$

$= \sqrt{3} \times \frac{435}{365} = 2.064$

$\phi = \tan^{-1}(2.064)$

$= 64.15°$

$PF = \cos\phi = 0.43$

iii) Reactive power $= \sqrt{3}\, V_L I_L \sin\phi$

$W_1 - W_2 = V_L I_L \sin\phi$

$W_1 - W_2 = 400 - (-35) = 435$

Reactive power $= \sqrt{3} \times 435 = 753.44$ VAR also, $W_1 - W_2 = V_L I_L \sin\phi$

but reactive power $Q = \sqrt{3} V_L I_L \sin\phi$

$$Q = \sqrt{3}(W_1 - W_2)$$

$$\frac{Q}{P} = \frac{\sqrt{3} V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

$$\Rightarrow \boxed{\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}}$$

power factor angle, $\phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$

Effect of power factor on wattmeter reading

$W_R = V_L I_L \cos(30° - \phi) = W_1$

$W_Y = V_L I_L \cos(30° + \phi) = W_2$

Case-i if $\phi = 0$, $W_1 = V_L I_L \cos 30°$

$W_2 = V_L I_L \cos 30°$

$$\boxed{W_1 = W_2}$$

if power factor angle is 0°, both wattmeter readings are equal.

Case-ii if $\phi = 30°$ $W_1 = V_L I_L$

$W_2 = \frac{V_L I_L}{2}$ $\boxed{W_2 = \frac{W_1}{2}}$

if power factor angle is 30°, one wattmeter readings is half of the other.

Case-iii if $\phi = 60°$

$W_1 = \frac{\sqrt{3}}{2} V_L I_L$, $W_2 = 0$

Transients

Steady state and transient response

→ Transients are regarded as sudden change in the state of a N/w or circuit which are generally indicated by switch operation (or) special i/p function starting at a particular instant of time.

→ A circuit having constant source is said to be in steady state if the currents & voltages donot change with time.

→ In a N/w containing energy storage elements (L&C), with change in excitation, the currents and voltages change from one state to other state.

→ The behaviour of the voltage or current when it is changed from one state to another is called the transient state. The time taken for the circuit to change from one steady state to another steady state is called the transient time.

→ Any circuit (or) N/w has 2 type of response

i) The response of a circuit or N/w with source present in it is called forced response which leads to steady state response.

ii) The response of any circuit or N/w without any source is called natural response which leads into transient response.

Total response = Forced response + Natural response

Forced response ↑ zero state response

Natural response ↑ zero i/p response

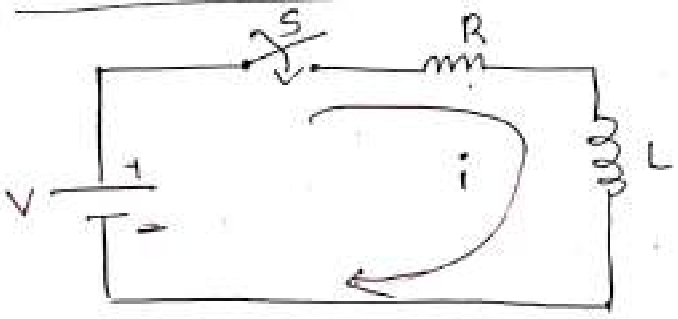
complete solution of Differential eqn = Particular integral + complementary function

ex: 1st order eqn $i(t) = 10 - 5e^{-2t}$

10 ↑ steady state response

$5e^{-2t}$ ↑ transient response

DC response of an R-L circuit



→ consider a circuit consisting of a resistance and inductance. The inductor in the circuit is initially uncharged and in series with the resistor.

→ When the switch 's' is closed, we can find the complete solution for the current.
Application of KVL to the circuit results in the following DE

$$V = Ri + L\frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

in the eq^n, current i is the solution to be found and voltage V is the applied constant voltage.

→ The voltage V is applied to the circuit only when the switch 'S' is closed.

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$$

The above equation is a linear differential equation of 1st order.

if $\frac{dy}{dx} + Ay = B$, then solution is given by

$$y = e^{-At} \int B e^{At} dt + c e^{-At}$$

C = Arbitary constant

Similary the current eq^n can be written as

$$i = e^{-\frac{R}{L}t} \int \frac{V}{L} e^{(R/L)t} + c e^{(-R/L)t}$$

$$i = \frac{V}{R} + c e^{-(R/L)t} \quad \text{.. (ii)}$$

To Findout the value of 'c', we use the initial conditions.

* inductor doesnot allow sudden changes in currents.

At $t=0$, $i=0$

$$0 = c + \frac{V}{R} \Rightarrow \boxed{c = -\frac{V}{R}}$$

substituting the value of 'c' in eqn (2), we get

$$i = \frac{V}{R} - \frac{V}{R} e^{-(R/L)t}$$

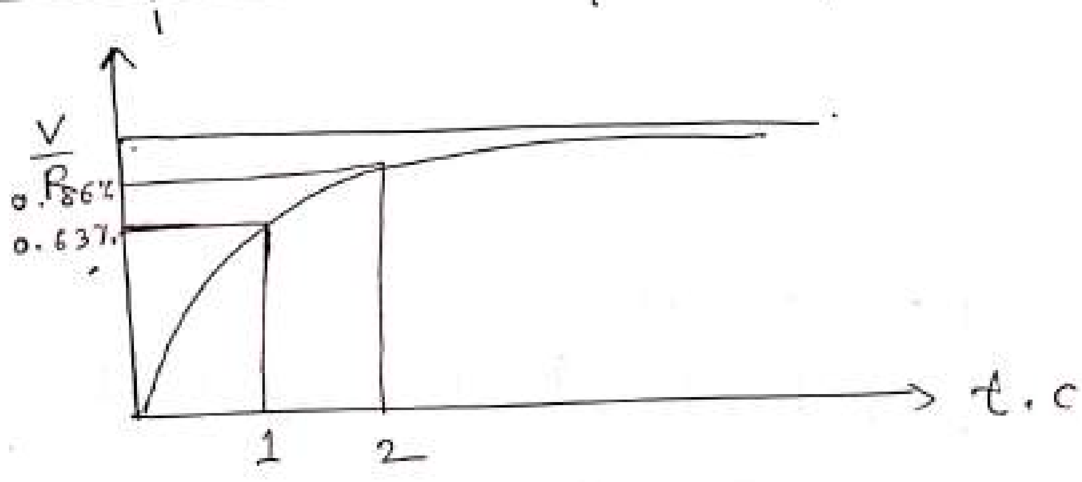
$$\Rightarrow \boxed{i = \frac{V}{R}\left[1 - e^{-(R/L)t}\right]} \quad i = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$$

$\frac{V}{R}$ = steady state part of eqn

$\frac{V}{R} e^{-(R/L)t}$ = transient part of eqn

→ when switch S is closed, the response reaches a steady state value after a time interval.

Time constant (τ) = $\frac{L}{R}$ $\quad i = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$



At $t = 0$, $i = 0$

At $t = \tau$, $i = 0.632\frac{V}{R} = 63.2\%$ of $\frac{V}{R}$ $\quad \boxed{\because e = 2.718}$

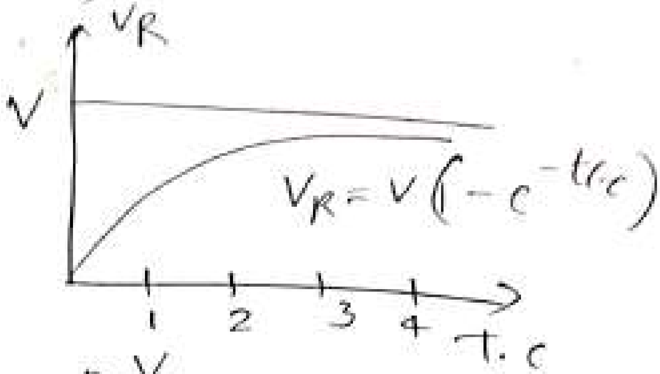
At $t = 2\tau$, $i = 0.86\frac{V}{R} = 86.4\%$ of $\frac{V}{R}$

At $t = 5\tau$, the transient part reaches 99% of its final value.

Voltage across the resistor (R)

$$V_R = iR = R \times \frac{V}{R}\left(1 - e^{-R/L\, t}\right)$$

$$V_R = V\left(1 - e^{-t/\tau}\right)$$

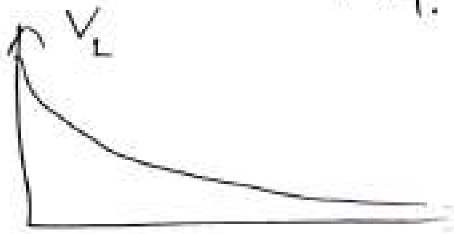


Voltage across inductor (V_L)

$$V_L = L\frac{di}{dt}$$

$$= L\,\frac{V}{R} \times \frac{R}{L}\, e^{\left(-R/L\right)t}$$

$$\boxed{V_L = V e^{-t/\tau}}$$

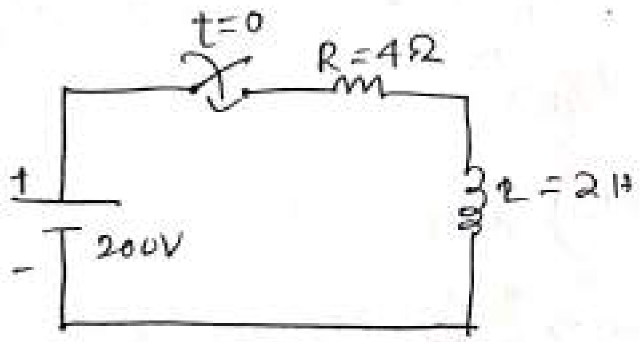


power in the resistor $P_R = V_R i_R$

power in the inductor $P_L = V_L i$

Q) In series R-L circuit, $R = 4\Omega$, $L = 2H$. Switch is closed at $t = 0$ sec. calculate i) transient current eqn
ii) steady state current
iii) time constant $V = 200V$
iv) Voltage across inductor & resistor?

A:



At $t = 0$, L is open circuited, $i = 0$ (initial condition)

a) Now transient current $i = \frac{V}{R}\left(1 - e^{-R/L\, t}\right)$

$$= \frac{200}{4}\left(1 - e^{-4/2\, t}\right) = 50\left(1 - e^{-2t}\right) \text{Amp}$$

(b) steady state current, $i = \frac{V}{R} = \frac{200}{4} = 50$ Amp

(c) Time constant (τ) $= \frac{L}{R} = \frac{2}{4} = 0.5$ sec

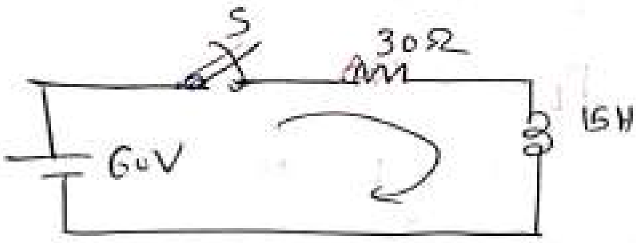
(d) Voltage across resistor (V_R) $= iR$

$$= V(1 - e^{-R/L\, t})$$

$$= 200(1 - e^{-2t}) \text{ volt}$$

(5) Voltage across inductor (V_L) $= Ve^{-2t} = 200\, e^{-2t}$ volt

Q) A series R-L circuit with $R = 30\Omega$, $L = 15H$ has a constant voltage $V = 60V$ applied at $t = 0$. Determine the current i, the voltage across resistor & the voltage across the inductor ?



A: by applying KVL, we get

$$15\frac{di}{dt} + 30i = 60 \Rightarrow \frac{di}{dt} + 2i = 4$$

$$i = \frac{V}{R}\left(1 - e^{-t/\tau}\right) \qquad \tau = \frac{L}{R} = \frac{15}{30} = \frac{1}{2} =$$

$$i = \frac{60}{30}\left(1 - e^{-t/(1/2)}\right)$$

$$i = 2\left(1 - e^{-2t}\right) \text{ Amp}$$

Voltage across resistor $V_R = iR$

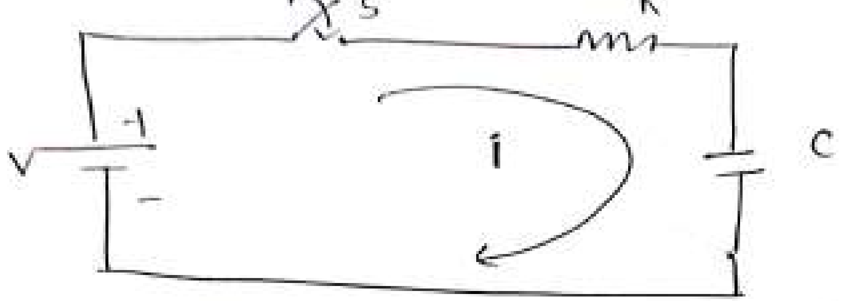
$$= 2(1 - e^{-2t}) \times 30$$

$$= 60(1 - e^{-2t}) \text{ (Ans)}$$

Voltage across inductor, $V_L = L\frac{di}{dt}$

or $Ve^{-2t} = 60e^{-2t}$ volt (Ans)

DC Response of an RC circuit



→ consider a circuit consisting of resistance (R), & capacitance. The capacitor is intially uncharged and is in series with a resistor.

→ when the switch S is closed at $t=0$,

Apply KVL in the circuit

$$V = iR + \frac{1}{C}\int i\,dt$$

by differentiating the above eqn, we get

$$0 = R\frac{di}{dt} + \frac{i}{C} \Rightarrow \boxed{\frac{di}{dt} + \frac{1}{RC}\, i = 0} \quad \text{..(1)}$$

eqn (1) is a linear differential eqn with only the complementary function.

Soln of eqn (1) will be

$$i = c\, e^{-t/RC}$$

to find the value of 'c', we use initial condn.

→ The capacitor never allow sudden changes in voltage; it will act as a short circuit at $t=0^+$.

At $t=0$, $i = \frac{V}{R}$

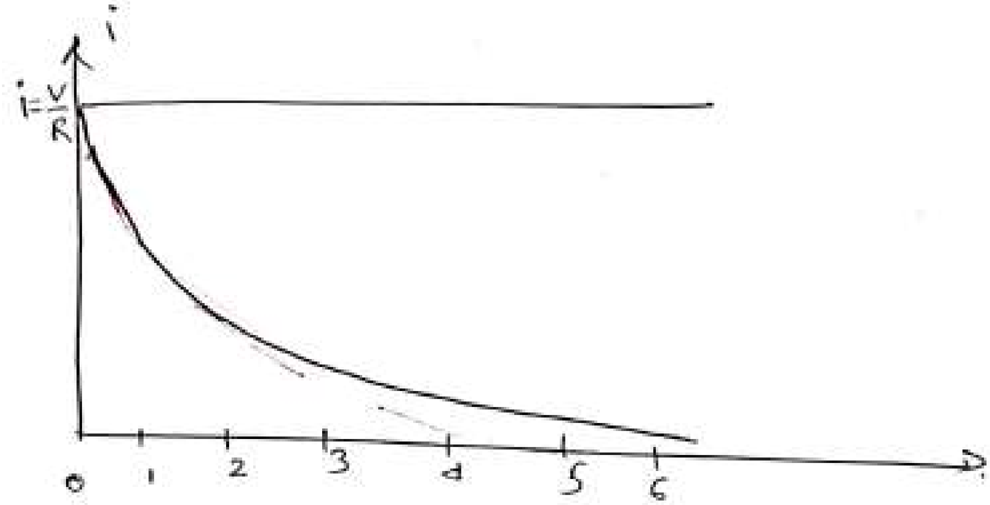
Substituting this current in eq we get

$\frac{V}{R} = c$

The current eqn becomes

$$i = \frac{V}{R} e^{t/RC}$$

Time constant ($\tau = RC$ sec)



~~Af~~

Voltage across the resistor is

$$V_R = Ri = R \times \frac{V}{R} e^{-(t/RC)t} \quad \therefore V_R = Ve^{-t/RC}$$

similarly the voltage across capacitor is

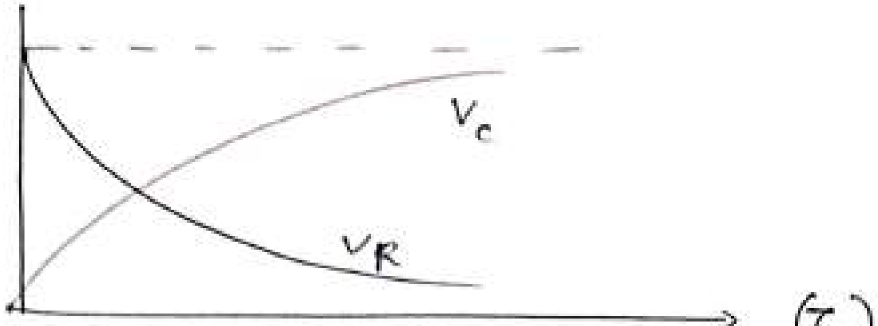
$$V_c = \frac{1}{c}\int i\,dt$$

$$= \frac{1}{C}\int \frac{V}{R} e^{-t/RC} dt$$

$$= -\left[\frac{V}{RC} \times RC\, e^{-t/RC}\right] + c$$

At $t=0$, voltage across capacitor is zero, $c = V$

$$\therefore V_c = V(1 - e^{-t/RC})$$



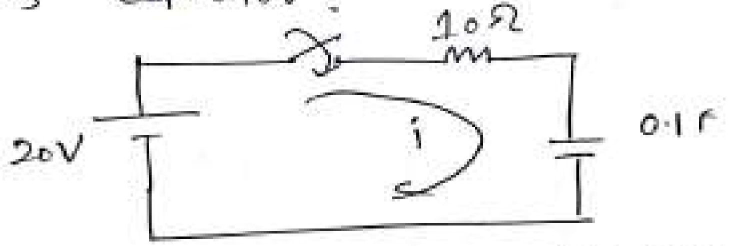
Power in the resistor, $P_R = V_R i$

$$= \frac{V^2}{R} e^{-2t/RC}$$

Power in the capacitor, $P_C = V_C i = \frac{V^2}{R}\left(e^{-t/RC} - e^{-2t/RC}\right)$

Q) A series RC circuit consists of a resistor of 10Ω & capacitor of 0.1F. A constant voltage of 20V is applied to the circuit at t=0. Obtain the current eq^n. Determine the voltages across the resistor & capacitor?

A:



By applying KVL, we get

$$10i + \frac{1}{0.1}\int i\,dt = 20$$

derivative on both sides

time constant (τ)

$RC = 10 \times 0.1$
$= 1$ sec

$$10\frac{di}{dt} + \frac{i}{0.1} = 0 \Rightarrow \frac{di}{dt} + i = 0$$

current eq^n $= i = \frac{V}{R} e^{-t/RC}$

$$= \frac{20}{10} e^{-t/1} = 2e^{-t}$$

Voltage across resistor is $V_R = iR = 2e^{-t} \times 10 = 20e^{-t}$

Voltage across capacitor, $V_C = V\left(1 - e^{-t/RC}\right)$

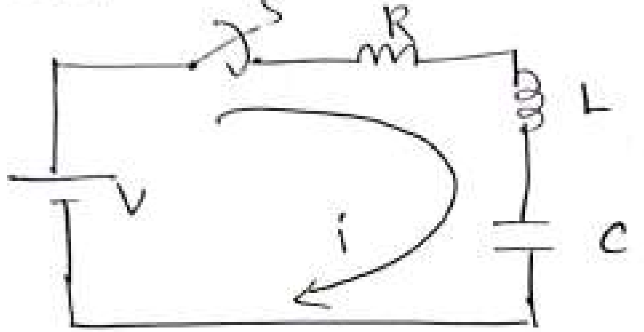
$$= 20\left(1 - e^{-t}\right) \text{ Volt.}$$

DC response of an R-L-C circuit

→ consider a circuit consisting of R, L & C.

→ The capacitor & inductor are initially uncharged and are in series with a resistor.

When switch 's' is closed at $t=0$



Applying KVL to the circuit

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

taking derivative on both sides

$$\Rightarrow 0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i$$

$$\Rightarrow \boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0} \quad \cdots (1)$$

The above eqn is a 2nd order linear differential eqn, with only complementary function.

characteristics eqn of the eqn (1) is

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right) = 0 \quad \cdots (ii)$$

roots of eqn (ii) $D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

Assume $k_1 = -\frac{R}{2L}$ & $k_2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

i.e. $D_1 = k_1 + k_2$, $D_2 = k_1 - k_2$

k_2 can be +ve, -ve or zero.

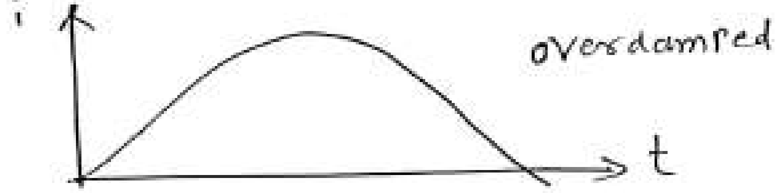
* k_2 is +ve, when $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

→ The roots are real & unequal, and give the over damped response. then eqn (1) becomes

$$\left[D - (k_1 + k_2)\right]\left[D - (k_1 - k_2)\right] i = 0$$

The solution for the above eqn is

$$i = c_1 e^{(k_1+k_2)t} + c_2 e^{(k_1-k_2)t}$$



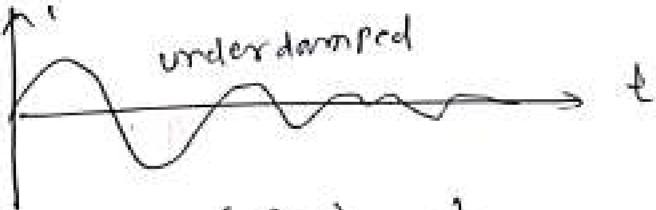
* k_2 is -ve, when $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

The roots are complex conjugate and give the underdamped response.

$$\left[D - (k_1 + jk_2)\right]\left[D - (k_1 - jk_2)\right] i = 0$$

The solution for the above eqn is

$$i = e^{k_1 t}\left[c_1 \cos k_2 t + c_2 \sin k_2 t\right]$$



* $k_2 = 0$ when $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

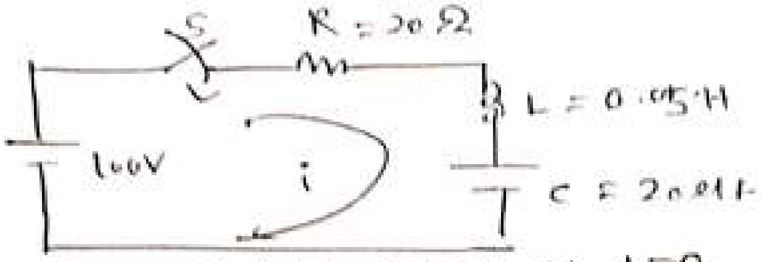
The roots are equal & give the critically damped case,

$$(D - k_1)(D - k_1)\, i = 0$$

The solution for the above eqn is

$$i = e^{k_1 t}(c_1 + c_2 t)$$

Q)



the switch is closed at t=0, find the transient current?

A→ At t=0, switch 's' is closed when the 100V source is applied to the circuit.

Applying KVL

$$100 = 20i + 0.05\frac{di}{dt} + \frac{1}{20\times10^{-6}}\int i\,dt$$

derivative on both sides

$$\Rightarrow 0.05\frac{d^2i}{dt^2} + 20\frac{di}{dt} + \frac{1}{20\times10^{-6}}i = 0$$

$$\Rightarrow \left(\frac{d^2i}{dt^2} + 400\frac{di}{dt} + 10^6 i\right) = 0$$

$$\Rightarrow (D^2 + 400D + 10^6)\,i = 0$$

$$D_1, D_2 = -200 \pm \sqrt{(200)^2 - 10^6}$$

$$D_1 = -200 + j\,979.8$$
$$D_2 = -200 - j\,979.8$$

$$i = e^{-200t}(c_1 \cos 979.8t + c_2 \sin 979.8t)\,A$$

$$\boxed{c_1 = 0,\ c_2 = 2.04}$$

$$\Rightarrow \boxed{i = e^{-200t}(2.04 \sin 979.8t)}\ \text{Amp}$$

Filters

i) A filter is a reactive Network that freely passes the desired bands of frequencies while almost totally suppressing all other bands.

ii) If a filter is designed with passive elements, it is called passive filter. if any active element is present, it is called active filter.

iii) Ideally, filter should produce no attenuation in the desired band, called the transmission band or pass band and should provide total or infinite attenuation at all other frequencies, called attenuation band or stop band.

iv) The frequency which separates the transmission band and the attenuation band is defined as the cut-off frequency of the wave Filters and is designated by f_c.

Classification of Filters

Filters are classified as follows

a) As per the relation between their arm impedances:

i) constant K-filter or prototype filter

ii) m-derived filter

b) As per their frequency characteristics

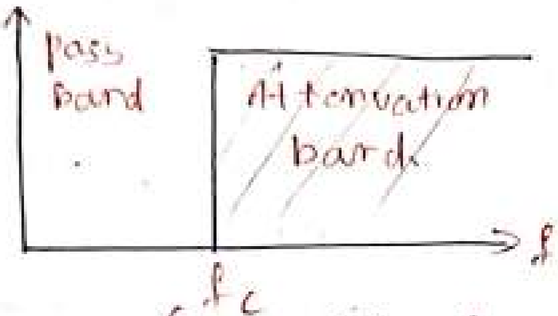
i) low-pass filter (LPF)

ii) High-pass filter (HPF)

iii) Band-stop filter (BPF)
iv) Band-pass filter (BSF)

Low Pass Filter (LPF)

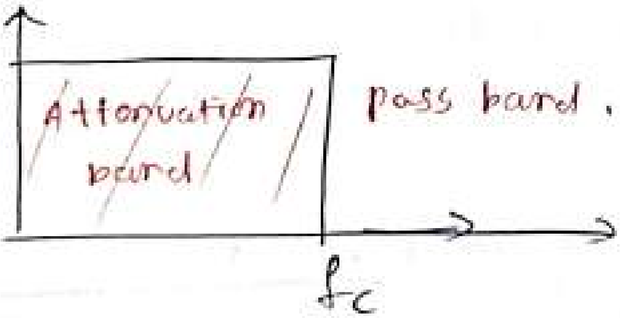
A low pass filter is one which passes without attenuation all frequencies upto the cut-off frequency (f_c) and attenuates all other frequencies greater than f_c.



(cut-off frequencies)

High pass Filter (HPF)

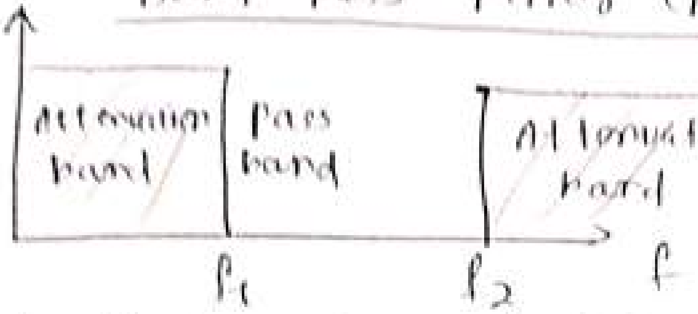
A high pass Filter attenuates all frequencies below a designated cut-off frequency (f_c) and passes all frequencies above f_c.



~~High pass Filter (HPF)~~

~~A high pass filter attenuates all frequencies below a designated cut-off frequency (f_c) and passes all frequencies above (f_c).~~

(c) Band Pass Filter (BPF) (3)



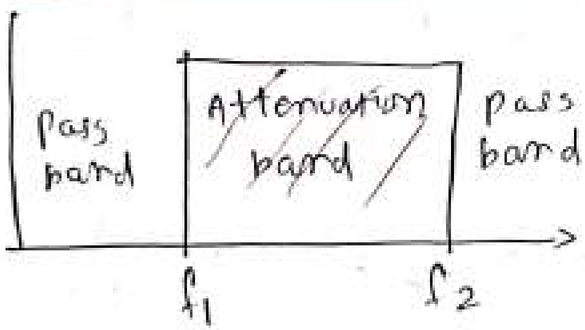
A band pass filter passes frequencies between two designated cut-off frequencies and attenuates all other frequencies.

$f_2 - f_1$ = band pass

f_1 = called lower cut-off frequency

f_2 = upper cut-off frequency

(d) Band Elimination Filter / band stop filter (BSF)



f_1 = lower cut-off frequency

f_2 = upper cut-off frequency

A band elimination filter passes all frequencies lying outside a certain range, while it attenuates all frequencies b/w two designated frequencies.

→ All fregencies b/w f_1 & f_2 will be attenuated while frequencies below f_1 and above f_2 will be passed.

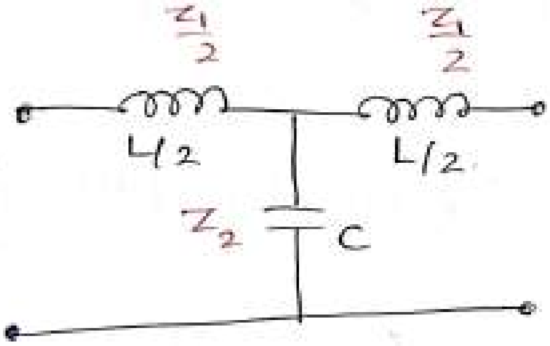
Constant-k low pass filter

(1)

→ A network, either T or π is said to be of the constant-k type if Z_1 and Z_2 of the N/W satisfy the relation $\boxed{Z_1 Z_2 = k^2}$

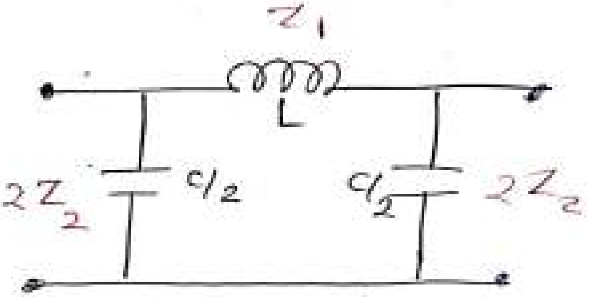
k = real constant, that is the resistance

→ The constant k, T or π type filter is also known as the prototype because other more complex Networks can be derived from it.



(a)

(T - N/w)



(π - N/w)

$$\boxed{\begin{aligned} Z_1 &= j\omega L \\ Z_2 &= \frac{1}{j\omega C} \end{aligned}}$$

∴ Hence $Z_1 Z_2 = j\omega L \times \frac{1}{j\omega C}$

$= \frac{L}{C} = k^2$

which is independent of frequency.

$Z_1 Z_2 = k^2 = \frac{L}{C}$

or $k = \sqrt{\frac{L}{C}}$

since the product Z_1 & Z_2 is constant, the filter is a constant-k type.

$$\Rightarrow \boxed{f_c = \frac{1}{\pi\sqrt{LC}}} \ \text{Hz}$$

$\boxed{K = \sqrt{\frac{L}{C}}}$ is called the design impedance or load resistance

$\boxed{C = \frac{1}{\pi f_c K}}$ gives the value of shunt capacitance

$\boxed{L = K^2 C = \frac{K}{\pi f_c}}$ gives the value of series inductance.

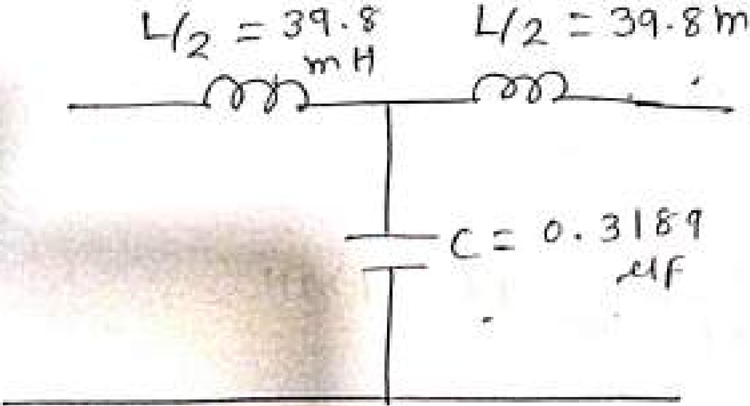
Q) design a low pass filter (both π and T-sections) having a cut-off frequency of 2KHZ to operate with a terminated load resistance of $500\,\Omega$.

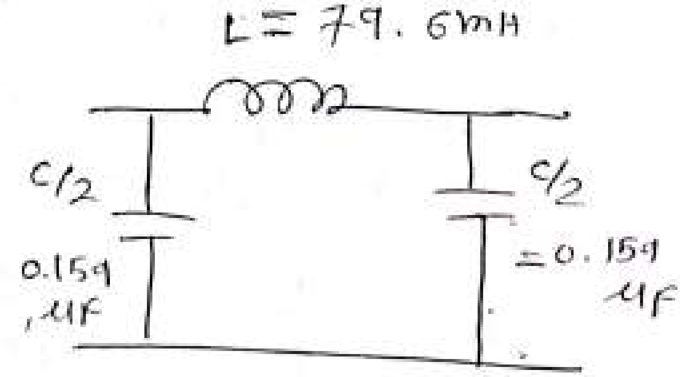
A: It is given that $K = \sqrt{\frac{L}{C}} = 500\,\Omega$

and $f_c = 2000\,\text{HZ}$

$$L = \frac{K}{\pi f_c} = \frac{500}{3.14 \times 2000} = 79.6\,\text{mH}$$

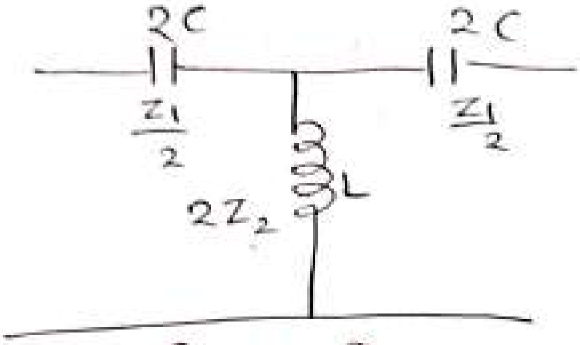
$$C = \frac{1}{\pi f_c K} = \frac{1}{3.14 \times 2000 \times 500} = 0.318\,\mu\text{F}$$



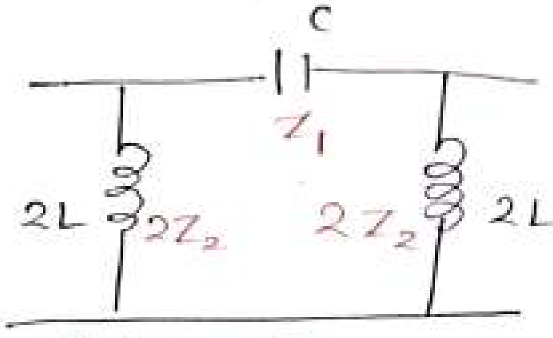


Constant K-high Pass Filter

→ Constant k-high pass filter can be obtained by changing the positions of series and shunt arms of the N/ws.



(T - N/w)



(π - N/w)

product of Z_1 & Z_2 is independent of frequency & the filter design obtained will be of the constant k-type. $Z_1 = -j/\omega C$ $Z_2 = j\omega L$

$$Z_1 Z_2 = \frac{-j}{\omega C} j\omega L = \frac{L}{C} = k^2 \qquad k = \sqrt{\frac{L}{C}}$$

The cut-off frequencies are given by $f_c = \frac{1}{4\pi\sqrt{LC}}$

$$L = \frac{k}{4\pi f_c} \text{ and } C = \frac{1}{4\pi f_c k} \qquad = \frac{k}{4\pi L} = \frac{1}{4\pi C k}$$

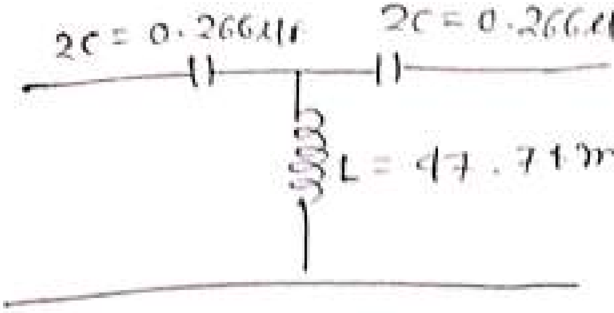
a) design a high pass filter having cut-off frequency of 1 KHZ with a load resistance of 500Ω.

A: Given $R_L = k = 600\Omega$

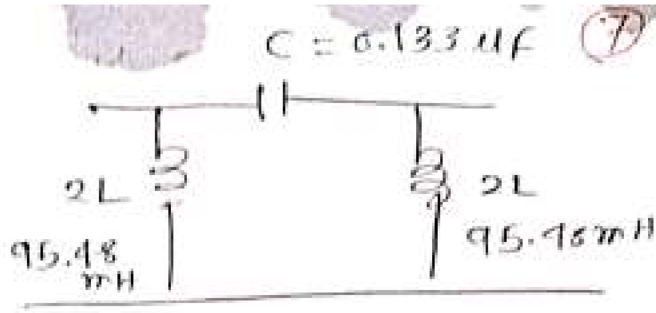
$f_c = 1000\,HZ$

$$L = \frac{k}{4\pi f_c} = \frac{600}{4 \times \pi \times 1000} = 47.77\,mH$$

$$C = \frac{1}{4\pi k f_c} = \frac{1}{4\pi \times 600 \times 1000} = 0.133\,\mu F$$

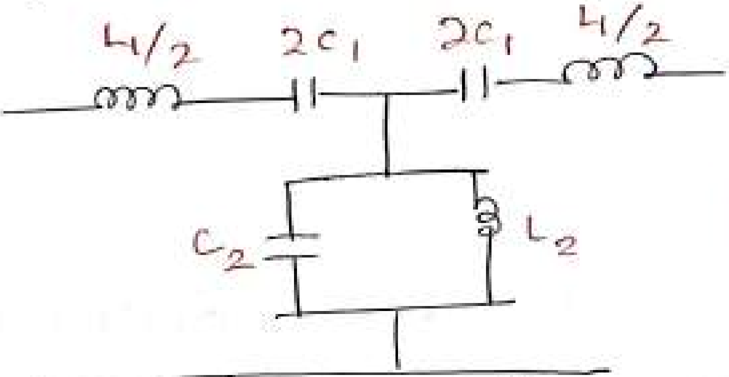


or

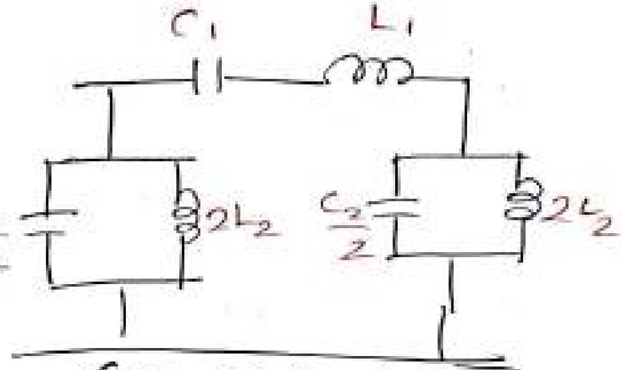


Band pass filter (BPF)

A band pass filter is one which attenuates all frequencies below a lower cut-off frequency f_1 & above an upper cut-off f_2.



(π - section)



(T-section)

$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = K^2$$

∵ K is a constant, hence the filter is a constant k-type.

$\rightarrow L_1 C_1 = L_2 C_2 = K^2$

$$\boxed{L_1 = \frac{K}{\pi(f_2 - f_1)}}$$

$$\boxed{C_1 = \frac{f_2 - f_1}{4\pi K f_1 f_2}}$$

$$\boxed{L_2 = \frac{(f_2 - f_1) K}{4\pi f_1 f_2} = C_1 K^2}$$

$$\boxed{C_2 = \frac{1}{\pi(f_2 - f_1)K} = \frac{L_1}{K^2}}$$

Q) design a k-type band pass filter having a design impedance of $500\,\Omega$ & cut-off frequencies 1 kHz and 10 kHz. (8)

A: $K = 500\,\Omega$, $f_1 = 1000$ Hz

$f_2 = 10000$ Hz

$$L_1 = \frac{k}{\pi(f_2 - f_1)} = \frac{500}{\pi \times 9000} = \frac{55.55}{\pi} \text{ mH} = 16.68 \text{ mH}$$

$$C_1 = \frac{f_2 - f_1}{4\pi K f_1 f_2} = \frac{9000}{4\pi \times 500 \times 1000 \times 10000} = 0.143\,\mu F$$

$L_2 = C_1 K^2 = 3.57$ mH

$C_2 = \frac{L_1}{k^2} = 0.0707\,\mu F$

each of the two series arms of the constant k, T-section filter is given by

$$\frac{L_1}{2} = \frac{17.68}{2} = 8.84 \text{ mH}$$

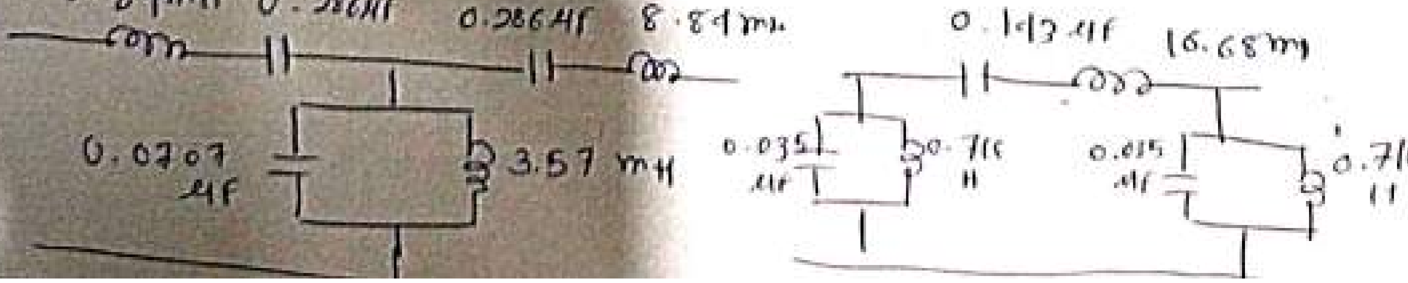
$2C_1 = 2 \times 0.143 = 0.286\,\mu F$

Shunt elements of the N/W are given by

$C_2 = 0.0707\,\mu F$ & $L_2 = 3.57$ mH

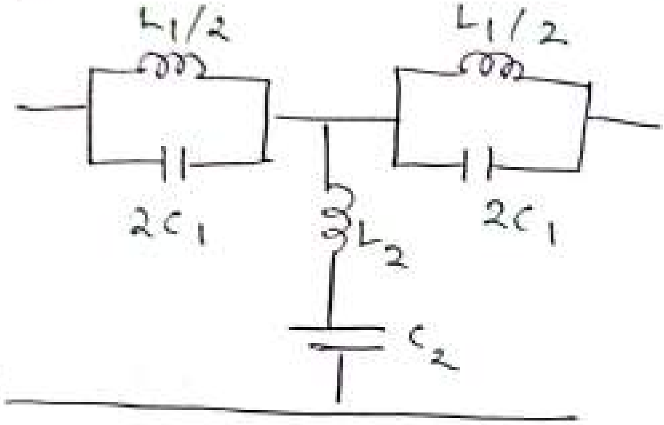
$C_1 = 0.143\,\mu F$, $L_1 = 16.68$ mH

$\frac{C_2}{2} = 0.035\,\mu F$, $2L_2 = 2 \times 0.0358 = 0.0716$ H

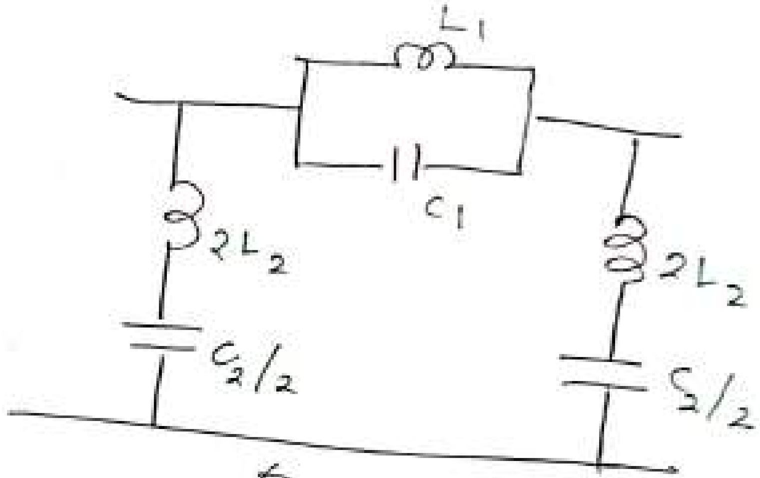


Band elimination filter / Band stop filter

A band elimination filter or band stop filter is one which passes without attenuation all frequencies less than lower cut-off frequency (f_1) and ~~upper~~ greater than upper cut-off frequency f_2.



(T-section)



(π-section)

$$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = K^2 \qquad f_0 = \sqrt{f_1 f_2}$$

$$\boxed{C_2 = \frac{1}{K\pi}\left|\frac{f_2 - f_1}{f_1 f_2}\right|}$$

$$\boxed{K^2 = \frac{L_1}{C_2} = \frac{L_2}{C_1}}$$

$$\boxed{L_2 = \frac{K}{4\pi(f_2 - f_1)}}$$

$$\boxed{L_1 = K^2 C_2 = \frac{K}{\pi}\left(\frac{f_2 - f_1}{f_1 f_2}\right)}$$

$$\boxed{C_1 = \frac{L_2}{K^2} = \frac{1}{4\pi K(f_2 - f_1)}}$$

Q) Design a band elimination filter having a design impedance of 600Ω & cut-off frequencies $f_1 = 2\,KHZ$ & $f_2 = 6\,KHZ$

Solution $f_1 = 2KHz$, $f_2 = 6KHz$ $\quad K = 600\Omega$

$f_2 - f_1 = 4KHz$

$$L_1 = \frac{K}{\pi}\left(\frac{f_2 - f_1}{f_2 f_1}\right) = \frac{600 \times 4000}{\pi \times 2000 \times 6000} = 63\ mH$$

$$C_1 = \frac{1}{4\pi K(f_2 - f_1)} = \frac{1}{4 \times \pi \times 600 \times 4000} = 0.033\ \mu F$$

$$L_2 = \frac{1}{4\pi K(f_2 - f_1)} = \frac{600}{4\pi \times 4000} = 12\ mH$$

$$C_2 = \frac{1}{K\pi}\left(\frac{f_2 - f_1}{f_1 f_2}\right) = \frac{1}{600 \times \pi}\left(\frac{4000}{2000 \times 6000}\right) = 0.176\ \mu F$$

each of the two-series arms of the constant K, T-section filter is given by

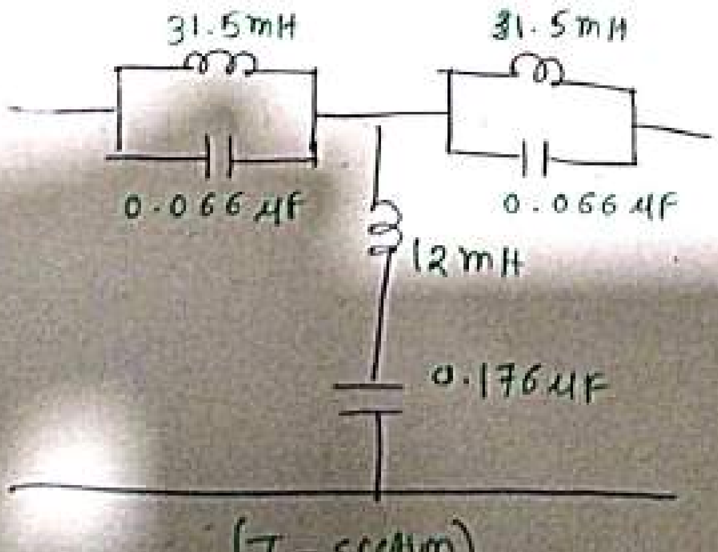
$\frac{L_1}{2} = 31.5\ mH$, $2C_1 = 0.066\ \mu F$

shunt arms of the N/ω $L_2 = 12\ mH$, $C_2 = 0.176\ \mu F$

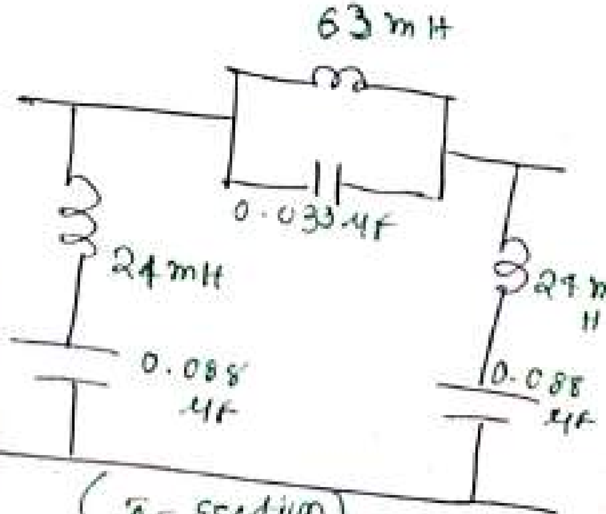
For the constant K, π-section filter the elements of the series arm are $L_1 = 63\ mH$, $C_1 = 0.033\ \mu F$

& the elements of the shunt arms are

$2L_2 = 24\ mH$, $\frac{C_2}{2} = 0.088\ \mu F$



(T - section)



(π - section)