

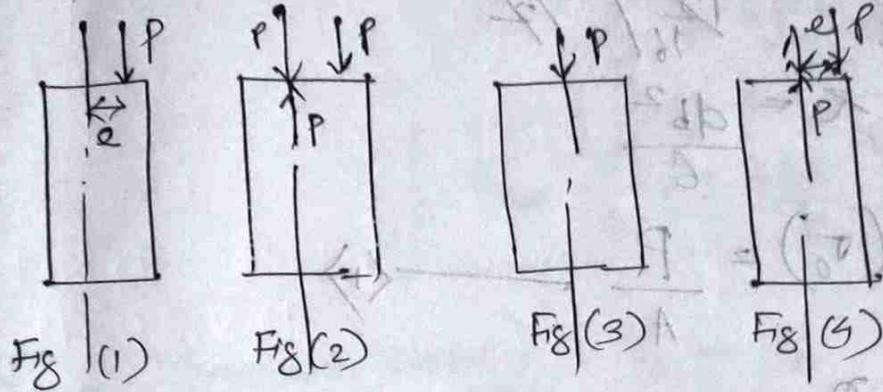
**A LECTURE NOTE
ON
TH 2- STRENGTH OF
MATERIAL
SEMESTER -3**



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Columns with Eccentric load :-



Fig(1) - The given load P , acting at an eccentricity of (e) .

Fig(2) - Put two equal and opposite forces (P) along the axis of strut.

Fig(3) - One of 3 forces acting along the axis of strut which cause direct stress.

Fig(4) - The other two forces will form a couple. The moment of this couple will be equal to $P \times e$ which produce a bending stress.

Maximum & Minimum stress

Let P = Eccentric load on the column

e = eccentricity of load

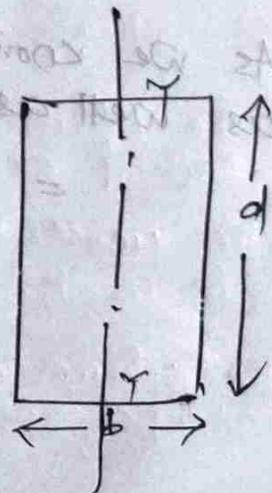
b = width of column

d = depth / thickness of column

Area of column section $A = b \times d$

Moment of inertia of column about

$Y-Y$ axis $I = \frac{db^3}{12}$



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From section modulus

$$Z = \frac{I}{Y} = \frac{db^3}{12} \cdot \frac{2}{b/2}$$

$$Z = \frac{db^2}{6}$$

Direct stress (σ_0) = $\frac{P}{A}$

from bending eqⁿ $\frac{M}{I} = \frac{\sigma_b}{Y}$

$$\sigma_b = \frac{M \times Y}{I}$$

$$\frac{M}{\left(\frac{I}{Y}\right)}$$

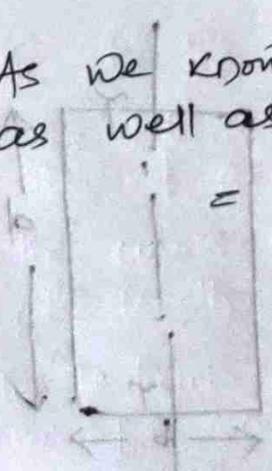
$$= \frac{P \times e}{\frac{db^2}{6}}$$

$$= \frac{6Pe}{db \cdot b}$$

$$= \frac{6Pe}{A \cdot b} \quad \text{②}$$

As we know eccentric load causes direct stress as well as bending stress, Thus total stress

$$= \sigma_0 \pm \sigma_b$$



$$\begin{aligned}\sigma_{\max} (\text{maximum stress}) &= \sigma_0 + \sigma_b \\ &= \frac{P}{A} + \frac{6Pe}{Ab}\end{aligned}$$

$$\boxed{\sigma_{\max} = \frac{P}{A} \left(1 + \frac{6e}{b} \right)}$$

$$\begin{aligned}\sigma_{\min} (\text{Minimum stress}) &= \sigma_0 - \sigma_b \\ &= \frac{P}{A} - \frac{6Pe}{A \cdot b}\end{aligned}$$

$$\boxed{\sigma_{\min} = \frac{P}{A} \left(1 - \frac{6 \cdot e}{b} \right)}$$

A rectangular column of width 120mm and of thickness 100mm carries a point load of 120kN at an eccentricity of 10mm. Determine the max & min stress at the base of column.

Data Given

$$b = 120 \text{ mm}$$

$$t = 100 \text{ mm}$$

$$P/W = 120 \text{ kN} = 120 \times 10^3 \text{ N}$$

$$e = 10 \text{ mm}$$

$$\sigma_{\max} = \frac{P}{A} \left(1 + \frac{e \cdot 6}{b} \right)$$

$$= \frac{120 \times 10^3}{120 \times 100} \left(1 + \frac{10 \times 6}{120} \right)$$

$$= 15 \text{ N/mm}^2$$

$$\begin{aligned} \sigma_{\min} &= \frac{P}{A} \left(1 - \frac{6 \cdot e}{b} \right) \\ &= \frac{180 \times 10^3}{120 \times 100} \left(1 - \frac{6 \times 10}{120} \right) \\ &= 5 \text{ N/mm}^2 \end{aligned}$$

Column

A member of structure or bar which carries an axial compressive load is called strut.

If the member of structure is vertical i.e. perpendicular or inclined at 90° to the surface/horizontal is known as column.

Short column :- Column which have lengths less than 8 times their respective diameters.

Long column :- Column which have lengths more than 30 times their respective diameters.

Slenderness Ratio (K) - It is the ratio of unsupported length of the column to the minimum radius of gyration of the cross-section.

Assumptions Made in Euler's Column Theory :-

- 1) The column is initially perfectly straight and the load is applied axially.
- 2) The cross-section of the column is uniform throughout its length.
- 3) The column material is perfectly elastic, homogeneous & isotropic and obey Hooke's Law.

- 4) The length of the column is very large as compared to its lateral dimensions.
- 5) The direct stress is very small as compared to the bending stress.
- 6) The column will fail by buckling alone will negligible.

Crippling/Buckling/Critical load:-

It is the load at which the column will start to bend, is known as crippling load.

(or)

It is the max compressive load of a column can withstand before it buckles or collapses.

According Euler's column theory

~~l = length of column~~
 ~~P = actual load~~

l = actual length of column

l_e = effective length

P = load on column

E = Young's Modulus

I = Moment of inertia

$$\text{Crippling load/Buckling load} = \frac{2\pi^2 EI}{l_e^2}$$

Effective Length (L_e)

Sr. No.	End Condition	Eff. Length	Euler's Formula
1	Both Ends Fixed	$L_e = L/2$	$P_E = \frac{4 \pi^2 E I_{min}}{L^2}$
2	One end is Fixed and Other End Hinged	$L_e = L/\sqrt{2}$	$P_E = \frac{2 \pi^2 E I_{min}}{L^2}$
3	Both Ends Hinged	$L_e = L$	$P_E = \frac{\pi^2 E I_{min}}{L^2}$
4	One end is Fixed and Other is free	$L_e = 2L$	$P_E = \frac{\pi^2 E I_{min}}{4L^2}$