A LECTURE NOTE
ON
TH 2- STRENGTH OF MATERIAL
SEMESTER -3

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Mechanical Engineering

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Theory of simple Bending :—

When external load acts on a beam, the shear force and bending moments are setup at all the section of beam.

Due to that SF & BM, the beam undergoes some deformation. The material of beam offers resistance or stress against the deformation. The stresses introduced by the bending moment is known as bending stresses.

Assumptions of Simple Bending :—

1) The material of beam is homogeneous and isotropic.
   - homogeneous → Same material
   - isotropic → same elastic property.

2) The value of young's modulus is same in the tension and compression.

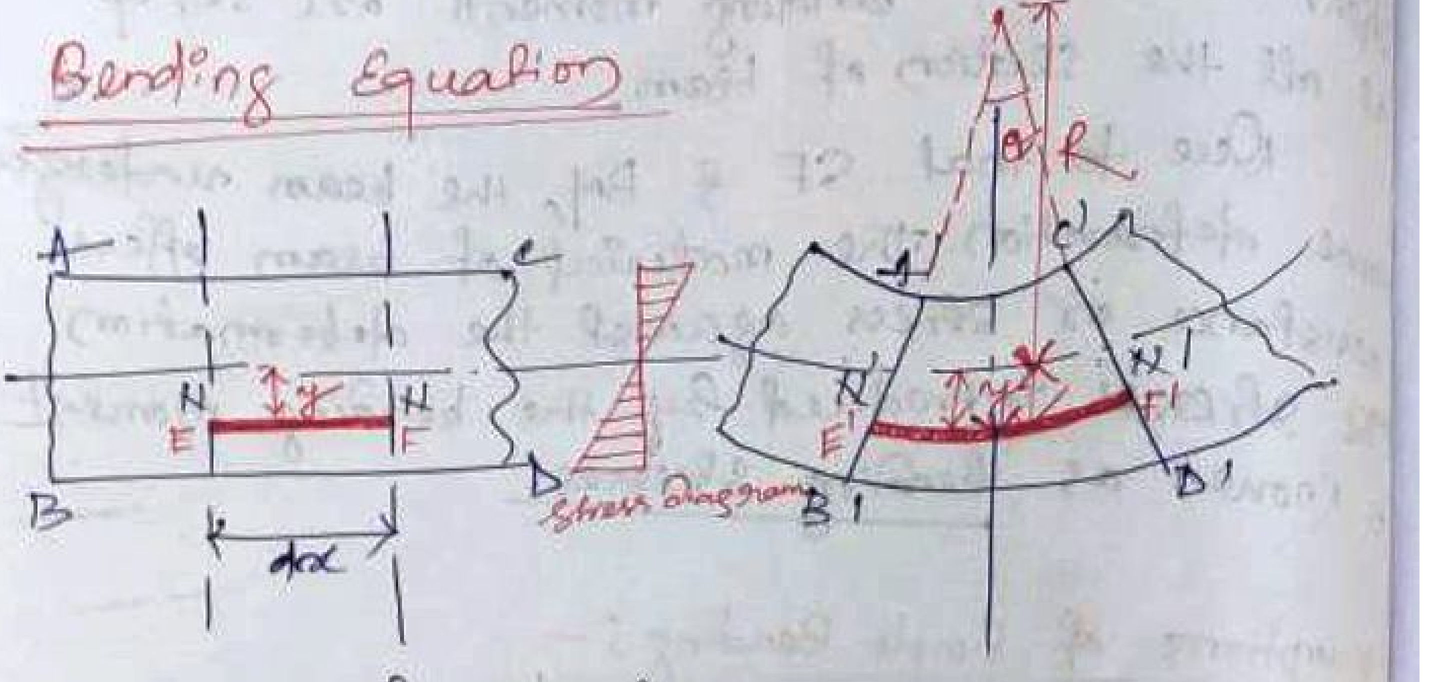
3) The transverse section which is plane before bending, remain plane after bending also.

4) The beam is initially straight & longitudinal filament bends into circular arcs with a common centre of curvature.

5) The radius of curvature large compared with the dimensions of cross-section.

6) Each layer of beam is free to expand or contract, independently of the layer above or below it.

Bending Equation



R = radius of curvature
θ = angle made by A'B' & C'D'

Consider a layer EF at a distance Y below the neutral axis (NN). after bending EF changes to E'F'.

original layer of EF = dx ——⟨1⟩
Length of neutral layer NN = dx
after bending N'N' = NN = dx ——⟨2⟩
also N'N' = R×θ ——⟨3⟩
E'F' = (R+y)×θ ——⟨4⟩

Hence $dx = R \times \theta$ —— (5).

Increased length of layer EF =

$$E'F' - EF = (R+y)\theta - R\theta$$
$$= y\theta \text{ —— (6)}$$

Strain in the layer (EF) =

$$= \frac{\text{Change in length}}{\text{Original length}}$$

$$e = \frac{y\theta}{R\theta}$$

$$e = \frac{y}{R} \text{ —— (7)}$$

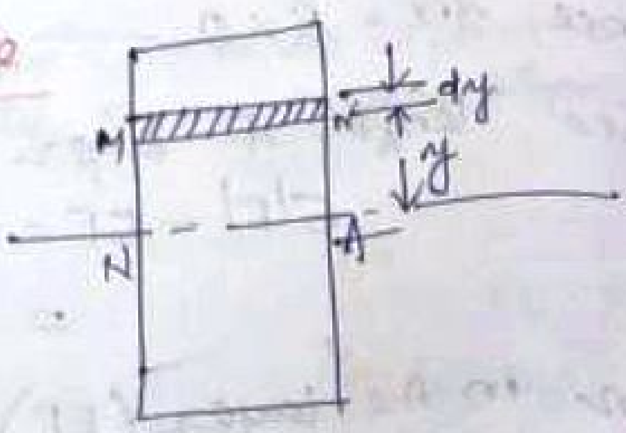
we know $E = \sigma / e$. (Stress/Strain)

$$E = \frac{\sigma}{(y/R)}$$

$$E \times \frac{y}{R} = \sigma$$

$$\boxed{\frac{E}{R} = \frac{\sigma}{y}} \text{ —— (8)}$$

Moment of Resistance



from eqⁿ (8) we get $\frac{\sigma}{y} = \frac{E}{R}$

or $\sigma = \frac{E}{R} \times y$

consider a small layer MN which is at a distance y from neutral axis of a area (dA).

Now the force of MN (dF) = stress × area

$$dF = \sigma \times dA$$

$$dF = \frac{E}{R} \times y \, dA$$

Total force on the beam

$$\int_0^F dF = \frac{E}{R} \int y \, dA \quad \text{——— (9)}$$

But pure bending, there is no force on the section of beam (force = 0)

$$\frac{E}{R}\int y\,dA = 0 \qquad \because \frac{E}{R} = \text{Constant}$$

$$\int y\,dA = 0$$

Force layer $= \frac{E}{R} \times y \times dA$

Moment about neutral axis = force × distance

$$M = \frac{E}{R} \times y \times dA \times y$$

$$M = \frac{E}{R} \times y^2 \times dA$$

Total moment $(M) = \frac{E}{R}\int y^2 dA$ ——— (10)

But we know $\int y^2 dA = I$ (moment of inertia)

So $M = \frac{E}{R} \times I$

or
$$\boxed{\frac{M}{I} = \frac{E}{R}} \quad \text{——— (11)}$$

from eqn (8) $\frac{E}{R} = \frac{\sigma}{y}$ so,

The eqn will be
$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}} \quad \text{— (12)}$$

Bending eqn

Neutral Axis (NA) :

The neutral axis of any transverse section of a beam is defined as the line of intersection of the neutral layer with the transverse section. It is written as N.A.

Section Modulus (Z) :

It is the ratio of moment of inertia of a section about the neutral axis to the distance of outermost layer from the neutral axis. It is denoted by symbol Z.

Mathematically $Z = \frac{I}{y}$

from eqn ⟨12⟩ $\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$.

$$M = \sigma_{max} \times \frac{I}{y_{max}}$$

$$\boxed{M = \sigma_{max} \times Z}$$