

**A LECTURE NOTE
ON
TH.3 – ENGINEERING
MATHEMATICS-I
SEMESTER -1**



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Mechanical Engineering

**GOVT. POLYTECHNIC,
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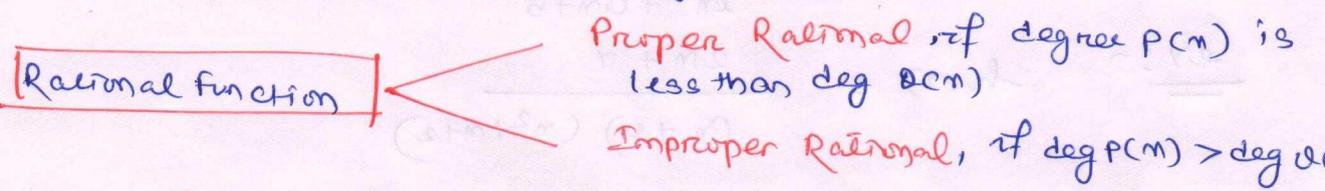
Partial fractions and Integral of Rational function:-

Defn:- Any function of the form $\frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are two different polynomials are called as 'Rational function'!

Generally, polynomials can be expressed like $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n$, $a_n \neq 0$ is said to be a polynomial of degree 'n'.

Example:-

- 1. $P(x) = x^3 + 3x^2$ — degree = 3 → **Cubic polynomial**
- 2. $P(x) = 2 = 2 \cdot x^0$ — degree = 0 → **constant**
- 3. $P(x) = x + 5$ — degree = 1 → **Linear**
- 4. $P(x) = x^2 + 2x + 5$ — degree = 2 → **Quadratic**



PROCEDURE TO BREAK A FRACTION INTO RATIONAL FRACTION:-

Step-1:- Consider a proper rational function ($\text{deg } P(x) < \text{deg } Q(x)$), then they can be expressed into simpler forms. So four different cases arises.

Case-1 Factorising the denominator into real factors, we get that non-repeated distinct linear factor arises. Then for each non-repeated linear factor, the corresponding partial fraction will be $\frac{A}{ax+b}$, if A is constant.

ex- $f(x) = \frac{2x}{x^2 + 5x + 6}$

On factorising $Q(x) = x^2 + 5x + 6$ we get that $x^2 + 5x + 6 = (x+3)(x+2)$

Non repeated, linear

∴ So the partial fraction will be

$$\frac{2x}{x^2 + 5x + 6} = \frac{A_1}{x+3} + \frac{A_2}{x+2}$$

A_1, A_2 are constant. we have to evaluate

check

$\text{deg } P(x) = 1$

$\text{deg } Q(x) = 2$

$P(x) = 2x$

$Q(x) = x^2 + 5x + 6$

∴ $\text{deg } P(x) = 1 <$

$\text{deg } (Q(x)) = 2$

⇒ Proper fraction

Case-II For any linear factor appearing 'r' times in the factorisation of $Q(x)$, the corresponding partial fractions will be sum $\frac{A_r}{(a+bx)^r}$, ($r=1, 2, 3, \dots$)

example

$$g(x) = \frac{2x+5}{(x+2)^3(x+5)}$$

So the partial fractions will be
 $\frac{2x+5}{(x+2)^3(x+5)}$
 3 times $\leftarrow \frac{(x+2)^3}{(x+2)^3}$ Repeated fraction $\frac{(x+5)}{(x+5)}$ Distinct

$$\frac{2x+5}{(x+2)^3} = \frac{A_1}{x+2} + \frac{A_2}{(x+2)^2} + \frac{A_3}{(x+2)^3} + \frac{A_4}{(x+5)}$$

Case-III For any non-repeated quadratic factor (x^2+ax+b) in the denominator, the corresponding partial fraction will be of the form $\frac{Ax+B}{x^2+ax+b}$

ex:- $h(x) = \frac{2x+7}{(x+2)(x^2+x+2)}$

$$= \frac{A}{x+2} + \frac{Bx+C}{x^2+x+2}$$

Case-IV To a repeated quadratic factor x^2+ax+b 'r' times, the corresponding pf are $\frac{A_r x + B_r}{(x^2+ax+b)^r}$, ($r=1, 2, \dots$)

ex:- $\frac{x+3}{(x^2+2x+5)^2} = \frac{a_1x+b_1}{x^2+2x+5} + \frac{a_2x+b_2}{(x^2+2x+5)^2}$

Step-I

Now evaluate the both side by making an identity of RHS and LHS.

Step-II

Comparing both side of equation and co-efficients of variable, we will get equalities, which will gives us the value of required constants.

Step-III

Substitute the various constant at their PFS, to get solution. Then proceed for integration.

Examples Based on the Partial Fraction Method; -

1) Integrate $\int \frac{x^2 - x - 2}{(x+1)(x-1)(x-2)} dx$.

Ans:- Clearly, it is a proper fraction with three distinct linear factors $x+1$, $x-1$, $x-2$. So the partial fraction will be

$$\frac{A_1}{x+1} + \frac{A_2}{x-1} + \frac{A_3}{x-2} = \frac{x^2 - x - 2}{(x+1)(x-1)(x-2)} \quad \text{--- (1)}$$

Taking LCM in RHS, we will get that

$$\therefore \frac{x^2 - x - 2}{(x+1)(x-1)(x-2)} = \frac{A_1(x-1)(x-2) + A_2(x+1)(x-2) + A_3(x+1)(x-1)}{(x-1)(x-2)(x+1)}$$

Canceling denominator of both side, we get

$$\begin{aligned} x^2 - x - 2 &= A_1(x-1)(x-2) + A_2(x+1)(x-2) + A_3(x+1)(x-1) \\ &= A_1(x^2 - 3x + 2) + A_2(x^2 - x - 2) + A_3(x^2 - 2) \\ &= x^2(A_1 + A_2 + A_3) + x(-3A_1 - A_2) + (2A_1 - 2A_2 - A_3) \end{aligned}$$

[\therefore we've collected coefficients of x^2 , x , constants from each factor.]

Now comparing the both sides,

$$x^2 = x^2(A_1 + A_2 + A_3) \Rightarrow A_1 + A_2 + A_3 = 1 \quad \text{--- (2)}$$

$$-x = x(-3A_1 - A_2) \Rightarrow -3A_1 - A_2 = -1 \quad \text{--- (3)}$$

$$-2 = 2A_1 - 2A_2 - A_3 \Rightarrow 2A_1 - 2A_2 - A_3 = -2 \quad \text{--- (4)}$$

\therefore solving (2), (3), (4) by elimination process,

we get that $A_1 = \frac{1}{6}$

$$A_2 = \frac{1}{2}$$

$$A_3 = \frac{1}{3}$$

\therefore Substituting the values of A_1, A_2, A_3 in eqn (1)

we get that

$$\therefore \int \frac{n^2 - n - 2}{(n+2)(n-1)(n-2)} dn = \int \frac{dn}{6(n+2)} + \int \frac{dn}{2(n-1)} + \int \frac{dn}{3(n-2)}$$

$$= \frac{1}{6} \ln|n+2| + \frac{1}{2} \ln|n-1| + \frac{1}{3} \ln|n-2| + c$$

Exercises Integrate the following

1. $\int \frac{dn}{n^2 - a^2}$

4. $\int \frac{3n}{(n-4)(n+2)} dn$

2. $\int \frac{dn}{a^2 - n^2}$

5. $\int \frac{5n-12}{(n+3)(n-1)} dn$

3. $\int \frac{4n-9}{n^2-5n+6} dn$

6. $\int \frac{4n+5}{n^2+n-2} dn$

Example - 2

Integrate $\int \frac{2n^2 + n + 3}{(n^2 + 2)(n-1)} dn$

Ans: clearly in denominator there is a quadratic term $(n^2 + 2)$ & linear factor $(n-1)$ appears. So the partial fraction will be

$$\frac{2n^2 + n + 3}{(n^2 + 2)(n-1)} = \frac{-An + B}{n^2 + 2} + \frac{C}{n-1} \quad \text{--- (1)}$$

Taking LCM of RHS and equating both sides, we get

$$= \frac{(-An + B)(n-1) + C(n^2 + 2)}{(n^2 + 2)(n-1)}$$

Cancelling denominator of both sides, we get that

$$\Rightarrow \frac{2n^2 + n + 3}{(n^2 + 2)(n-1)} = \frac{An^2 + Bn - An - B + Cn^2 + 2C}{(n^2 + 2)(n-1)}$$

$$\Rightarrow 2n^2 + n + 3 = (A+C)n^2 + (B-A)n + (2C-B)$$

Comparing the terms involving n^2 , n , const. from both side

$$2n^2 = (A+C)n^2 \Rightarrow 2 = A+C \quad \text{--- (2)}$$

$$n = (B-A)n \Rightarrow 1 = B-A \quad \text{--- (3)}$$

$$3 = 2C-B \Rightarrow 3 = 2C-B \quad \text{--- (4)}$$

$$\text{eq}^{\circ}(2) + 2\text{eq}^{\circ}(3) \Rightarrow$$

$$A + c + B - A = 2 + 2$$

$$\Rightarrow c + B = 3 \quad \text{--- (4)}$$

from (4) & (5)

$$2c - B + c + B = 3 + 3$$

$$\Rightarrow 3c = 6 \Rightarrow \boxed{c = \frac{6}{3} = 2}$$

from (2),

$$A + c = 2$$

$$\Rightarrow A + 2 = 2$$

$$\Rightarrow \boxed{A = 0}$$

from (3),

$$B - A = 2$$

$$\Rightarrow B - 0 = 2$$

$$\Rightarrow \boxed{B = 2}$$

\(\therefore\) Putting the values of A, B, c in eq^{\circ}(1), we get that

$$\begin{aligned} \int \frac{2x^2 + x + 3}{(x^2 + 2)(x - 2)} dx &= \int \frac{dx}{x^2 + 2} + \int \frac{2 dx}{x - 2} \\ &= \int \frac{dx}{x^2 + 2} + 2 \int \frac{dx}{x - 2} \\ &= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + 2 \ln|x - 2| + c \quad \text{D.} \end{aligned}$$

$$\frac{dx}{x^2 + a^2}$$

$$= \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{dx}{x + a}$$

$$= \ln|x + a| + c$$

Exercise

$$(1) \int \frac{2x^2 + x - 4}{(x^2 + 1)(x - 2)} dx$$

$$(2) \int \frac{4x^2 - x + 3}{(x^2 + 2)(x - 2)} dx$$

$$(3) \int \frac{x}{(x - 2)(x^2 + 4)} dx$$

$$(4) \int \frac{2x + 9}{(x + 3)^2} dx$$

$$(5) \int \frac{dx}{x^2 - 5}$$

$$(6) \int \frac{e^x}{e^{2x} + 3e^x + 2} dx$$

$$(7) \int \frac{x^2}{(x^2 + 2)(x^2 + 3)} dx$$

DEFINITE INTEGRATION

If $f(x)$ is any continuous function in $[a, b]$, then $\int_a^b f(x) dx$ is called the definite integral between the limit a and b .

$a \rightarrow$ lower / inferior limit

$b \rightarrow$ upper / superior limit

Fundamental Theorem of Integral Calculus:-

If $f(x)$ is continuous function in $[a, b]$ and $F(x)$ is anti-derivative of $f(x)$, then

$$\int_a^b f(x) dx = F(b) - F(a)$$

example

$$\begin{aligned} 1) \int_0^1 x^2 dx &= \left| \frac{x^3}{3} \right|_0^1 = \left(\frac{x^3}{3} \right)_{x=1} - \left(\frac{x^3}{3} \right)_{x=0} \\ &= \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3} - 0 = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} 2) \int_0^1 \frac{dx}{1+x^2} &= \tan^{-1} x \Big|_0^1 = \tan^{-1}(1) - \tan^{-1}(0) \\ &= \frac{\pi}{4} - 0 = \frac{\pi}{4} \end{aligned}$$

Exercises

1. $\int_0^4 [x] dx$

2. $\int_0^2 e^x dx$

3. $\int_0^2 e^{5x+3} dx$

10. $\int_0^{\pi/2} \frac{dx}{1+\cos x}$

13. $\int_0^{\pi/4} \frac{e^z}{e^{2z}+4} dz$

16. $\int_0^2 \frac{dx}{\sqrt{1-x^2}}$

4. $\int_{-1}^2 x^4 dx$

5. $\int_0^2 x^{1/3} dx$

6. $\int_1^4 \frac{dx}{\sqrt{x}}$

11. $\int_0^{\pi/4} \cos^2 x dx$

14. $\int_{-4}^3 |x| dx$

7. $\int_0^1 (2x+2)^4 dx$

8. $\int_0^{\pi/4} \cos 2x dx$

9. $\int_0^{\pi/2} (\cos x - 8 \sin x) dx$

12. $\int_0^{\pi/4} \sin^5 x \cdot \cos x dx$

15. $\int_{-2}^2 (2x-1)(x-2) dx$

Elementary Properties Of Definite Integral :-

$$1. \int_a^b f(x) dx = - \int_b^a f(x) dx.$$

By altering the upper & lower limits, the definite integral is multiplied by minus one.

$$\begin{aligned} \int_a^b f(x) dx &= F(b) - F(a) \\ &= - \left\{ F(a) - F(b) \right\} \quad (\because \text{Taking minus outside}) \\ &= - \int_b^a f(x) dx \end{aligned}$$

$$2. \int_a^b f(x) dx = \int_a^b f(y) dy = \int_a^b f(z) dz$$

Definite Integral is independent of the symbol of variable.

In each of the above case, the result will be $F(b) - F(a)$.

3. If x is in the interval $[a, b]$, there is a point ' k ' exist such that value at ' k ' ^{can be} found, then

$$\int_a^b f(x) dx = \int_a^k f(x) dx + \int_k^b f(x) dx.$$

$$\Rightarrow F(b) - F(a) = F(k) - F(a) + F(b) - F(k)$$

$$\Rightarrow F(b) - F(a) = F(b) - F(a)$$

Example:- Find the definite integral of [By property 3]

$$\int_{-3}^4 |x| dx = \int_{-3}^0 |x| dx + \int_0^4 |x| dx$$

(breaking interval so that positive values in one integral & negative values of ' x ' in another

$$= \int_{-3}^0 -x dx + \int_0^4 x dx = \int_0^4 x dx + \int_{-3}^0 x dx \quad (\text{By property 1})$$

we know $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ of ' x '

$$\begin{aligned}
&= \int_0^4 x \, dx + \int_0^{-3} x \, dx \\
&= \left. \frac{x^2}{2} \right|_0^4 + \left. \frac{x^2}{2} \right|_0^{-3} \\
&= \frac{4^2}{2} - \frac{0}{2} + \frac{(-3)^2}{2} - \frac{0}{2} \\
&= \frac{16}{2} - 0 + \frac{9}{2} - 0 \\
&= \frac{16}{2} + \frac{9}{2} = \frac{25}{2} \quad \text{D.}
\end{aligned}$$

Property-4

$$\int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx.$$

Proof:- Consider $\int_0^a f(a-x) \, dx$.
 By taking $a-x = z$, $dz = -dx$.
 So for $x=0$, $z = a-0 = a$
 for $x=a$, $z = a-a = 0$

$$\begin{aligned}
\therefore I &= \int_a^0 f(z) (-dz) \\
&= \int_a^0 -f(z) \, dz \\
&= \int_0^a f(z) \, dz \\
&= \int_0^a f(x) \, dx \quad \text{D.}
\end{aligned}$$

$$\int_0^a f(x) \, dx = \int_a^0 -f(x) \, dx$$

(By property - 1)

(By property - 2)
 Definite Integral is independent of symbol of variable)
 So we can use 'x' in place of z.

Property - 5

$$\int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx & \text{if } f(x) \text{ is even} \\ 0 & \text{if } f(x) \text{ is odd.} \end{cases}$$

Proof:-

Any function is said to be even function if it satisfies

$$f(x) = f(-x)$$

Let $y = x^2 = f(x)$

Then $f(-x)$ is the value by putting $-x$.

$$\therefore f(-x) = (-x)^2 = x^2 = f(x)$$

\therefore So $f(x)$ is even

Some even functions are $\cos x$, x^{2n} .

Any function $f(x)$ is said to be an odd function, if it satisfies

$$f(-x) = -f(x)$$

Some odd functions are $\sin x$, $\tan x$, x^n where n is odd.

Let $f(x) = \tan x$

$$\begin{aligned} \text{Put } -x, f(x) &= \tan(-x) = \frac{\sin(-x)}{\cos(-x)} \\ &= \frac{-\sin x}{\cos x} \\ &= -\tan x \\ &= -f(x) \end{aligned}$$

$\Rightarrow \tan x$ is an odd function. \square

$$\textcircled{1} \int_{-a}^a f(m) dm = \int_{-a}^0 f(m) dm + \int_0^a f(m) dm. \quad \text{--- } \textcircled{1}$$

$$= I_1 + I_2$$

for I_1 , take $m = -u \Rightarrow dm = -du$

$$\text{for } m = -a, u = a$$

$$m = 0, u = 0$$

$$\Rightarrow I_1 = \int_a^0 f(-u) - du = - \int_a^0 f(-u) du$$

$$= \int_0^a f(-u) du$$

$$= \int_0^a f(m) dm$$

So eqⁿ (1) becomes,

$$\int_{-a}^a f(m) dm = \int_{-a}^0 f(m) dm + \int_0^a f(m) dm.$$

If f is even, $f(-m) = f(m)$, so it reduces

$$= \int_0^a f(m) dm + \int_0^a f(m) dm$$

$$= 2 \int_0^a f(m) dm$$

Now if f is odd, then $f(-m) = -f(m)$, then

$$= \int_0^a -f(m) dm + \int_0^a f(m) dm$$

$$= \int_0^a f(m) dm - \int_0^a f(m) dm$$

$$= 0$$

D.

ex-1 Integrate $\int_{-\pi/2}^{\pi/2} \sin^3 x dx$

Ans:- As we know that $\sin x$ is odd function, so by property

$$\int_{-a}^a f(x) dx = 0 \quad \text{if } f(x) \text{ is odd}$$

$$= 2 \int_0^a f(x) dx, \quad \text{if } f(x) \text{ is even.}$$

ex-2 $\int_{-\pi/4}^{\pi/4} \cos^3 x dx$

Ans:- As we know that $\cos x$ is even function, then

$$\int_{-\pi/4}^{\pi/4} \cos^3 x dx = 2 \int_0^{\pi/4} \cos^3 x dx.$$

$$= 2 \int_0^{\pi/4} \cos^2 x \cdot \cos x dx.$$

$$= 2 \int_0^{\pi/4} (1 - \sin^2 x) \cos x dx.$$

Take $z = \sin x, dz = \cos x dx.$

for $x=0, z=0$; $x=\pi/4, z=1/\sqrt{2}$

$$= 2 \int_0^{1/\sqrt{2}} (1 - z^2) dz$$

$$= 2 \left\{ \int_0^{1/\sqrt{2}} dz - \int_0^{1/\sqrt{2}} z^2 dz \right\}$$

$$= 2 \left\{ z \Big|_0^{1/\sqrt{2}} - \frac{z^3}{3} \Big|_0^{1/\sqrt{2}} \right\}$$

$$= 2 \left\{ \frac{1}{\sqrt{2}} - \frac{2}{6\sqrt{2}} \right\}$$

$$= \frac{2}{\sqrt{2}} - \frac{2}{6\sqrt{2}} = \frac{5\sqrt{2}}{6} \quad \text{D.}$$

ex-3 Integrate $\int_0^{\pi/4} \log(1 + \tan x) dx$

Ans:- Let $I = \int_0^{\pi/4} \ln(1 + \tan x) dx$

$$= \int_0^{\pi/4} \ln(1 + \tan(\pi/4 - \theta)) d\theta$$

$$= \int_0^{\pi/4} \ln \left[1 + \frac{1 - \tan \theta}{1 + \tan \theta} \right] d\theta$$

using the property

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$\begin{aligned}
 &= \int_0^{\pi/4} \ln \left(\frac{2}{1+\tan \theta} \right) d\theta \\
 &= \int_0^{\pi/4} \ln 2 \, d\theta - \int_0^{\pi/4} \ln(1+\tan \theta) \, d\theta \\
 &= \int_0^{\pi/4} \ln 2 \, d\theta - I \\
 \therefore I &= \ln 2 \int_0^{\pi/4} d\theta - I \\
 &= \ln 2 \left[\theta \right]_0^{\pi/4} - I \\
 &= \frac{\pi \ln 2}{4} - I \\
 \Rightarrow 2I &= \frac{\pi \ln 2}{4} \\
 \Rightarrow I &= \frac{\pi \ln 2}{8}
 \end{aligned}$$

Example-3

Integrate $\int_0^{\pi/2} \frac{2 \, d\theta}{2+\tan \theta}$

Sol:-

$$\begin{aligned}
 \text{Let } I &= \int_0^{\pi/2} \frac{d\theta}{2+\tan \theta} \\
 &= \int_0^{\pi/2} \frac{d\theta}{2+\frac{\sin \theta}{\cos \theta}} \\
 &= \int_0^{\pi/2} \frac{\cos \theta \, d\theta}{2\cos \theta + \sin \theta} \quad \text{--- (1)} \\
 &= \int_0^{\pi/2} \frac{\cos(\frac{\pi}{2}-\theta)}{\cos(\frac{\pi}{2}-\theta) + \sin(\frac{\pi}{2}-\theta)} \, d\theta \\
 &= \int_0^{\pi/2} \frac{\sin \theta}{\sin \theta + \cos \theta} \, d\theta \quad \text{--- (2)}
 \end{aligned}$$

Adding (1) & (2) we get

$$\begin{aligned}
 2I &= \int_0^{\pi/2} \frac{\cos \theta \, d\theta}{\cos \theta + \sin \theta} + \int_0^{\pi/2} \frac{\sin \theta \, d\theta}{\sin \theta + \cos \theta} \\
 &= \int_0^{\pi/2} \frac{\cos \theta + \sin \theta}{\cos \theta + \sin \theta} \, d\theta \\
 &= \int_0^{\pi/2} 1 \, d\theta = \int_0^{\pi/2} d\theta = \theta \Big|_0^{\pi/2} = \frac{\pi}{2} \\
 \Rightarrow I &= \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}
 \end{aligned}$$

Exercise

$$Q1 > \int_0^{\pi/2} \frac{\sqrt{\sin m}}{\sqrt{\sin m} + \sqrt{\cos m}} dm$$

$$Q2 > \int_0^1 \frac{\ln(1+m)}{1+m^2} dm$$

$$Q3 > \int_0^{\pi} \frac{m \sin m}{1 + \sin m} dm$$

$$Q4 > \int_0^{\pi/2} \frac{2}{2 + \cot m} dm$$

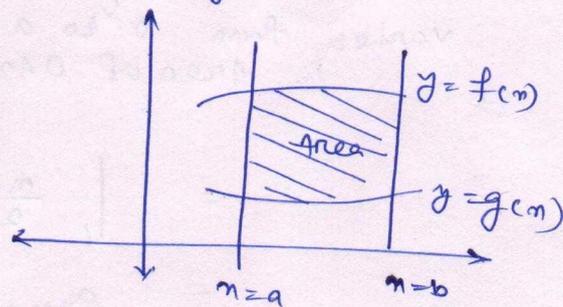
Geometrical Significance Of Definite Integration:-

The definite integral $\int_a^b f(m) dm$ represents the area under the curve $y = f(m)$, above the m -axis between the ordinates $m = a$ and $m = b$.

Area Between Two Curves:-

If there are two curves $y = f(m)$ and $y = g(m)$ where $g(m) < f(m)$ in $[a, b]$, then the area under the curve is given by

$$\begin{aligned} A &= \int_a^b f(m) dm - \int_a^b g(m) dm \\ &= \int_a^b [f(m) - g(m)] dm \end{aligned}$$



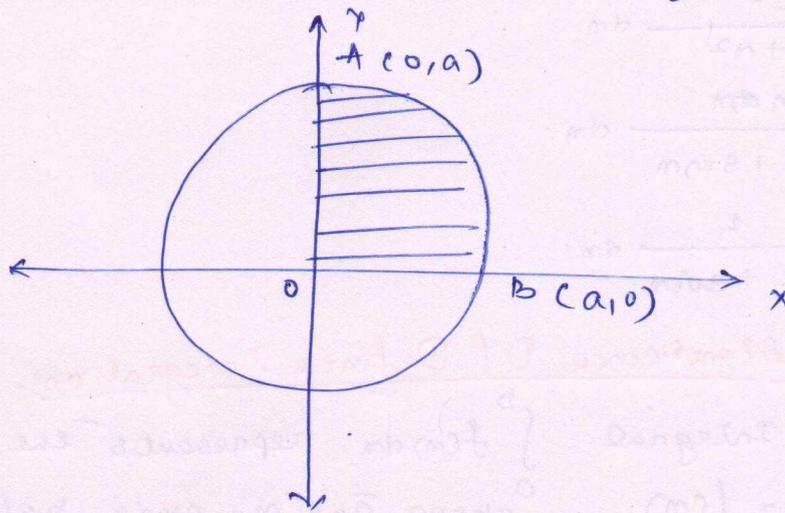
Example Find the area of the region enclosed by $y = 9 - m^2$, $y = 0$ and the ordinates $m = 0$ and $m = 2$ is given by.

Ans: The area under the curve $y = 9 - m^2$ bounded by $m = 0$ and $m = 2$ is given by

$$\begin{aligned} A &= \int_0^2 (9 - m^2) dm \\ &= \int_0^2 9 dm - \int_0^2 m^2 dm \\ &= 9m \Big|_0^2 - \frac{m^3}{3} \Big|_0^2 \\ &= 18 - \frac{8}{3} = \frac{46}{3} \text{ sq unit} \end{aligned}$$

2) Find the area of the circle $x^2 + y^2 = a^2$

Ans:- As we know that circle is symmetrical about y -axis. i.e. each semi circle have an area twice of its quadrant portion.



Total area of circle = $4 \times$ Area of region OAB.

Now $x^2 + y^2 = a^2$

$\Rightarrow y^2 = a^2 - x^2$

$\Rightarrow y = \sqrt{a^2 - x^2}$ in the 1st quadrant. Also 'x' varies from 0 to a along a x -axis.

1. Area of OAB = $\int_0^a \sqrt{a^2 - x^2} dx$

= $\left| \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right|_0^a$

= $\frac{a}{2} \times 0 + \frac{a^2}{2} \sin^{-1}(1) - 0 - 0$

= $\frac{a^2 \cdot \pi}{4} = \frac{\pi a^2}{4}$ sq unit.

Now Area of the circle = $4 \times$ Area of OAB region.

= $4 \times \frac{\pi a^2}{4} = \pi a^2$ sq unit. \square

Exercise

Q1) Determine the area bounded by the curve $y^2 = x$, $x=0$, $y=2$.

2018 (W)

Q2) Find the area bounded by the curve $xy = c^2$, the x -axis, $x=2$, $x=3$

2019 (S)

2019 (S)

Q3) Find the area bounded by the curve $y = c \cos x$, $y = b \sin x$ and $x=0$?